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FLUCTUATIONS AND CORRELATIONS IN SIMPLICIAL QUANTUM GRAVITY

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Recent results for simplicial quantum gravity in the Regge formulation are discussed. While no phase transition is found for pure gravity in two dimensions, a phase transition between a 'rough' and a 'smooth' phase of spacetime is found in both three and four dimensions. The importance of calculating a variety of geometric coordinate invariant correlation functions, in order to determine the properties of the lowest lying excitation, is emphasized. The nature of the apparently non-trivial fixed point found in the four-dimensional simplicial model for pure gravity is discussed. While fluctuations in the local volumes are responsible for critical behavior in two dimensions, in three and four dimensions it is instead the fluctuations in the local curvature that diverge at the critical point.

1. Introduction

It is generally agreed that the quantization of gravity represents one of the outstanding problems in theoretical physics, yet only limited progress has been achieved in understanding non-perturbative effects in the continuum. The lattice formulation of gravity provides a unique opportunity for addressing some of the unresolved questions like the nature of the ground state, properties of correlations, and large distance geometric properties. One the other hand some of the ambiguities present in the continuum theory persist in all known lattice formulations, namely the unboundedness of the pure gravitational Einstein-Hilbert action, the perturbative non-renormalizability and the non-uniqueness of the measure. We refer to references [1–4] for a discussion of these issues in the continuum and the lattice, and for a more adequate set of references.

Three models for lattice gravity exists, the simplicial quantum gravity models based on the Regge formulation (on a regular or on a dynamical random lattice), the hypercubic lattice model constructed in analogy to Wilson's lattice gauge formulation, and the dynamical triangulation model, which is a special case of Regge's model with equilateral triangles. The first two models are already classically completely equivalent to gravity, while the third model is known to be equivalent to gravity at the quantum level in two dimensions. The classical equivalence to gravity of the first two models makes the correspondence between lattice and continuum operators transparent, while the discreteness of the degrees of freedom in the third models allows them to be solved exactly (at least in certain cases) in two dimensions. Since all lattice models classically break reparametrization invariance in one way or another, it appears rather remarkable that recent results in the Regge and dynamical triangulation models seem to give similar results for the phase diagram of gravity in two, three and four dimensions. Of course such indications are not entirely unexpected on the basis of universality, by the apparent restoration of coordinate invariance at large distances (well documented at least in two dimensions) is still very encouraging. Indeed it has been suggested by some authors that the lattice approach is more fundamental.

Some important issues that need to be addressed in lattice quantum gravity include the restoration of coordinate invariance at large distances, the possible phases of the theory, the nature of universality in lattice gravity, and the properties of the lowest lying excitations in the various phases. Here I will discuss briefly some of the more recent results, as well as some topics related to the nature correlations in quantum gravity, which are relevant to the issue of showing that the graviton actually emerges as the only surviving large distance excitation. For a recent more in-depth review see reference [4].

A detailed description of the construction of the action for simplicial lattice gravity without and with matter fields can be found in the literature; in a number of cases the correspondence with the continuum answer can be established quite rigorously and in great generality. Given reasonable geometric and positivity properties of the action (and measure), universality is expected to lead to the same results for universal quantities (like physical observables, exponents and anomalous dimensions) in some continuum limit. Hopefully the physical particle spectrum will be independent of specific details of the ultraviolet lattice cutoff and the specific form of the lattice action, as long as it preserves the basic symmetries, at least approximately in some smooth limit.

In four dimensions the lattice action [5,6]

$$I_{latt} = \sum_{\text{hinges h}} \left[\lambda \, V_h - k \, \delta_h A_h + a \, \frac{A_h^2 \delta_h^2}{V_h} \right] \qquad (1)$$

is being used. Here δ_h is the deficit angle at a hinge (triangle), A_h is the area of the hinge, and V_h is the volume associated with that hinge. Classically the above action is equivalent to the continuum higher-derivative action

$$I_{cont} = \int d^4x \, \sqrt{g} \left[\lambda - \frac{k}{2} \, R + \frac{a}{4} \, R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \right] \qquad (2)$$

with a cosmological constant term (proportional to λ), the Einstein-Hilbert term ($k = 1/8\pi G$), and a higher-derivative term with dimensionless coupling constant a^{-1}. As usual, the lattice and continuum actions differ by higher order lattice correction terms (higher derivative terms), which should become irrelevant in the continuum limit. Quantum mechanically it is known that the continuum limit has to be taken at a phase transition point, i.e. at a non-trivial fixed point of the renormalization group, if one can be found. At the fixed point the correlation length diverges and the relevant excitations then extend over many lattice spacings, making the specific lattice details fade away.

Quantities of physical interest which can be computed include the average volume $\mathcal{V}$, the average curvature $\mathcal{R}$, and their respective fluctuations χ_V and $\chi_{\mathcal{R}}$. In four dimensions one finds a clear transition separating a 'smooth' (small negative average curvature) from a 'rough' (very large positive average curvature) phase of spacetime [5,6]. One finds in the 'smooth' phase (for $\lambda = 1$)

$$\begin{aligned} \mathcal{R}(k) \quad &\sim_{k \to k_c} A_{\mathcal{R}} \, (k_c - k)^{\delta} \\ \chi_{\mathcal{R}}(k) \; &\sim_{k \to k_c} A_{\chi} \, (k_c - k)^{\delta - 1} \end{aligned} \qquad (3)$$

where δ is a critical exponent (in d=4 $\delta = 0.60(6)$, while in d=3 $\delta = 0.81(5)$; close to d=2 one expects $\delta \sim 2/(d-2)$). Thus the results are consistent with the picture of a vanishing curvature and a divergent curvature fluctuation, at the same value of k_c. At this point one cannot entirely exclude a discontinuous transition at k_c, but the first-order value for δ (according to the Nauenberg-Nienhuis criterion) is excluded by several standard deviations.

In the 'rough' phase ($k > k_c$) the curvature is very large, and the simplices collapse into degenerate configurations with very small volumes. This is also the region of the usual weak-field expansion ($G \to 0$), and the results for the average curvature $\mathcal{R}$ are not inconsistent with known perturbative results within this expansion. Setting $k_c = c\Lambda^2$, where c is a constant independent of k, and Λ the ultraviolet cutoff, one obtains from Eq. (3)

$$\mathcal{R}(G) \sim A_{\mathcal{R}} \left(\frac{-1}{8\pi G} \right)^{\delta} \left[1 + \delta c \Lambda^2 (-8\pi G) + \cdots \right] \quad (4)$$

One can see that $\mathcal{R}$ is possibly not analytic at $G = 0$, and an expansion in powers of G involves increasingly higher powers of the ultraviolet cutoff Λ.

It is quite remarkable that if one computes the volume susceptibility χ_V instead, one finds that it approaches a finite value at k_c, suggesting the absence of critical volume fluctuations. These appear to be desirable properties in a quantum theory of gravity, where the excitations in the continuum are expected to be massless gravitons, without massless scalar states and without massless volume density fluctuations. This situation should be contrasted to the two-dimensional case, where the volume fluctuations (the Liouville mode) are found to be massless, as expected from continuum arguments.

Indeed in the harmonic gauge one has for the graviton propagator

$$k D_{\mu\nu\alpha\beta}(p) = \frac{4 P^{(2)}_{\mu\nu\alpha\beta}}{p^2 + \frac{2a}{k} p^4} + \frac{2 P^{(0)}_{\mu\nu\alpha\beta}}{-p^2 + \frac{a}{k} p^4} \quad (5)$$

and therefore

$$< \sqrt{g}(x) \sqrt{g}(y) > \sim \int dp \; e^{-ip(x-y)} \frac{P^{(0)}}{-p^2 + \frac{a}{k} p^4}$$

$$< \sqrt{g} R(x) \sqrt{g} R(y) > \sim \int dp \; e^{-ip(x-y)} \int dq \;\; \times$$

$$\tilde{R} \; \frac{4P^{(2)}}{p^2 + \frac{2a}{k} p^4} \; \tilde{R} \; \frac{4P^{(2)}}{(p-q)^2 + \frac{2a}{k}(p-q)^4} \quad (6)$$

and hence no propagation for the volume density fluctuations.

If one assumes that the curvature $\mathcal{R}$ has scaling dimension d_R, $\mathcal{R} \sim m^{d_R}$, then one obtains from Eq. (3)

$$\mathcal{R}(k) \underset{k \to k_c}{\sim} (k_c - k)^{\delta} \sim m^{d\delta/(1+\delta)} \quad (7)$$

In this model therefore the average curvature appears to be related to a dynamical 'graviton mass' via

$$\frac{m^2}{\mathcal{R}} \sim \left(\frac{\mathcal{R}}{\Lambda^2} \right)^{[2+(2-d)\delta]/d\delta} \quad (8)$$

with a calculable constant of proportionality of order unity.

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