Name: ___________

Final Exam

Physics 9

Problem I.

Compute numerically the following integral

$$\int_0^1 dx \int_0^1 dy \int_0^1 dz \, \frac{1}{(1+x^2+y^2+z^2)^2} \quad . \tag{1}$$

Can you determine (guess) what the exact answer is?

Problem II.

Solve the nonlinear equation for the pendulum,

$$\frac{d^2\theta(t)}{dt^2} \; + \; \frac{g}{l} \, \sin\theta(t) = 0 \tag{2}$$

for $g/l = 1$, and for times t up to 20. For initial conditions choose $\theta(0) = \pi/2$ and $\theta'(0) = 0$. Plot the solution and compare it with the solution of the equation valid for *small* oscillations,

$$\frac{d^2\theta(t)}{dt^2} \; + \; \frac{g}{l} \, \theta(t) = 0 \quad . \tag{3}$$

Discuss briefly your results.

Problem III.

Edward Lorenz introduced in 1961 a set of simple equations for modeling convection in the atmosphere. To his surprise, he found that the solutions can exhibit rather complex behavior. You are asked to solve the set of equations for the *Lorenz attractor*

$$
\begin{aligned}
x' &= -3\,(x-y) \\
y' &= -x\,z + 26.5\,x - y \\
z' &= x\,y - z
\end{aligned}
\tag{4}
$$

with initial conditions $x(0) = 0$, $y(0) = 1$, $z(0) = 0$, and for times up to $t = 50$. You should use the 4-th order *Runge-Kutta* integration method, which can be found in the package `<<Programm‘RungeKutta‘`. After loading the package, read the relevant description (using ?). Note that you only need to supply it with the *r.h.s.* of the equations.

You will need to supply a time step h, and you should take $h = 0.01$. In order to graph the resulting solution, the sequence of commands `Show[ Graphics3D[ Line[ % ] ]` can be used. You should look up these commands before you use them, to make sure you understand what they do. I suggest you first get the program running correctly with a short total integration time interval (like 1). Discuss briefly your observations of the solution.

Problem IV.

Generate a two-dimensional contour plot of the field

$$\phi(x, y) = \frac{1}{\sqrt{x^2 + y^2 + \epsilon}} \tag{5}$$

with $\epsilon = 0.1$ (which prevents the field from becoming singular at the origin). Restrict x and y to be in the interval $[-2, 2]$.

The *gradient* of a function $\phi(x, y, z)$ is defined as the vector constructed out of the spatial derivatives of ϕ,

$$\left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z} \right) \quad . \tag{6}$$

After loading the packages `<<Graphics‘PlotField‘` and `<<Graphics‘PlotField3D‘` you will be able to use the functions `PlotGradientField[ ]` and `PlotGradientField3D[ ]`. Look up their syntax. Then plot *minus* the gradient of the above $\phi(x, y)$ in two dimensions, and do the same for the function

$$\varphi(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2 + \epsilon}} \quad . \tag{7}$$

The resulting plots describe the electric field configurations corresponding to the above potentials.