SCALING DIMENSIONS OF LATTICE QUANTIZED GRAVITY

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I discuss a model for quantized gravitation based on the simplicial lattice discretization. It has been studied in detail using a comprehensive finite size scaling analysis combined with renormalization group methods. The results are consistent with a value for the universal critical exponent for gravitation $\nu = 1/3$, and suggest a simple relationship between Newton's constant, the gravitational correlation length and the observable average space-time curvature. Some perhaps testable phenomenological implications are discussed, such as the scale dependence of Newton's constant and properties of quantum curvature fluctuations.

1   Introduction

One of the outstanding problems in physics is a determination of the quantum-mechanical properties of Einstein's theory of Gravitation. Approaches based on linearized perturbation methods have had limited success so far, as the underlying theory is known not to be perturbatively renormalizable. Since gravitational fields are themselves the source for gravitation already at the classical level, perturbative results are perhaps of doubtful validity for sufficiently strong effective couplings. This is especially true in the quantum domain, where large fluctuations in the gravitational field appear at short distances. In general nonperturbative effects can give rise to novel behavior in quantum field theory, and in particular to the emergence of non-trivial fixed points of the renormalization group, a phase transition in statistical mechanics language. Then the universal low energy behavior of field theories is almost completely determined by the fixed point structure of the renormalization group trajectories [1].

The situation described above clearly bears some resemblance to the theory of strong interactions, Quantum Chromodynamics, where non-linear effects are known here to play an important role, and end up restricting the validity of perturbative calculations to the high energy, short distance regime [2]. For low energy properties the lattice formulation, combined with renormalization group methods and computer simulations, has provided convincing evidence for quark confinement and chiral symmetry breaking, two phenomena which are invisible to any order in the usual perturbative expansion.

2   The Lattice Theory

A discrete lattice formulation can be applied to the problem of quantizing gravitation; instead of continuous metric fields, one deals with new gravitational degrees of freedom, the edges, which live only on discrete space-time points and interact locally with each other. In Regge's simplicial formulation of quantum gravity [3] one then approximates the functional integration over continuous metrics by a dis-

cretized sum over piecewise linear simplicial geometries [4,5]. In such a model the role of the continuum metric is played by the edge lengths of the simplices, while local curvature is described by a set of deficit angles, determined via known formulae as functions of the given edge lengths. The simplicial lattice formulation of gravity is locally gauge invariant [6] and can be shown to contain perturbative gravitons in the lattice weak field expansion [7], making it an attractive and faithful lattice regularization of the continuum theory. In the end the original continuum theory of gravity is to be recovered as the space-time volume is made large and the fundamental lattice spacing of the discrete theory is sent to zero, without having to rely, at least in principle, on any further approximation to the original continuum theory.

Quantum fluctuations in the underlying geometry are represented in the discrete theory by fluctuations in the edge lengths, which can be modeled by a well-defined, and numerically exact, stochastic process. In analogy with other field theory models studied by computer, calculations are usually performed in the Euclidean imaginary time framework, which is the only formulation amenable to a controlled numerical study, at least for the immediate foreseeable future. The Monte-Carlo method, based on the concept of importance sampling, is well suited for evaluating in a numerically exact way the discrete path integral for gravity and for computing the required averages and correlation functions. By a careful and systematic analysis of the lattice results, the critical exponents can be extracted, and the scaling properties of invariant correlation functions determined from first principles [10].

The starting point for a non-perturbative study of quantum gravity is a suitable definition of the discrete Feynman path integral. In the simplicial lattice approach one starts from the discretized Euclidean path integral for pure gravity, with the squared edge lengths taken as fundamental variables,

$$
Z_L = \int_0^\infty \prod_s \left(V_d(s)\right)^\sigma \prod_{ij} dl_{ij}^2 \Theta[l_{ij}^2] \exp\left\{-\sum_h \left(\lambda V_h - k\,\delta_h A_h + a\,\delta_h^2 A_h^2/V_h\right)\right\} \ .
$$

(1)

The above expression represents a lattice discretization of the continuum Euclidean path integral for pure quantum gravity

$$
Z_C = \int \prod_x \left(\sqrt{g(x)}\right)^\sigma \prod_{\mu \geq \nu} dg_{\mu\nu}(x) \exp\left\{-\int d^4x\,\sqrt{g}\left(\lambda - \frac{k}{2}\,R + \frac{a}{4}\,R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right\} \ ,
$$

(2)

(with $k^{-1} = 8\pi G$, and G Newton's constant) and reduces to it for smooth enough field configurations. The discrete gravitational measure in Z_L should be considered as the lattice analog of the DeWitt continuum functional measure. The δA term in the lattice action is the well-known Regge term [3], and reduces to the Einstein-Hilbert action in the lattice continuum limit [7]. A cosmological constant term is needed for convergence of the path integral for large edge lengths, while the curvature squared term allows one to control the fluctuations in the curvature [4].

In general in the Regge theory the correspondence between lattice and contin-

uum operators is [4]

$$\sqrt{g}\,(x) \;\rightarrow\; \sum_{\text{hinges } h \supset x} V_h$$

$$\sqrt{g}\,R(x) \;\rightarrow\; 2 \sum_{\text{hinges } h \supset x} \delta_h A_h$$

$$\sqrt{g}\,R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}(x) \;\rightarrow\; 4 \sum_{\text{hinges } h \supset x} (\delta_h A_h)^2 / V_h \tag{3}$$

where δ_h is the deficit angle at the hinge (triangle) h, A_h is the area of hinge labelled by h, and V_h is the four-volume associated with the same hinge h.

In practice and for obvious phenomenological reasons one is only interested in the limit when the higher derivative contributions are small compared to the rest, $a \rightarrow 0$. In this limit the theory depends, in the absence of matter and after a suitable rescaling of the metric, only on one bare parameter, the dimensionless coupling k^2/λ. Without loss of generality, one can therefore set the bare cosmological constant $\lambda = 1$. In the following discussion only the case $a = 0$ will be considered.

3 Phases of Lattice Gravity

Studies on small lattices suggest a rich scenario for the ground state of quantum gravity [4,8,9]. The present evidence indicates that simplicial quantum gravity in four dimensions exhibits a phase transition (in the bare coupling G) between *two phases*: a strong coupling phase, in which the geometry is smooth at large scales and quantum fluctuations in the gravitational field eventually average out and are bounded; and a weak coupling phase, in which the geometry is degenerate and space-time collapses into a lower-dimensional manifold, bearing some physical resemblance to a spiky branched polymer. Only the smooth, small negative curvature (anti-DeSitter like) phase appears to be physically acceptable.

Phrased in a different language, the two phases of quantized gravity can loosely be described as having in one phase (with bare coupling $G < G_c$, the branched polymer-like phase) [9]

$$\langle g_{\mu\nu} \rangle \;=\; 0 \;, \tag{4}$$

while in the other (with bare coupling $G > G_c$, the smooth phase),

$$\langle g_{\mu\nu} \rangle \;\approx\; c\,\eta_{\mu\nu} \;, \tag{5}$$

with a vanishingly small negative average curvature in the vicinity of the critical point at G_c.

The existence of a phase transition at finite coupling G, usually associated in quantum field theory with the appearance of an ultraviolet fixed point of the renormalization group, implies in principle non-trivial, calculable non-perturbative scaling properties for correlations and effective coupling constants, and in particular here for Newton's gravitational constant. Since only the smooth phase with $G > G_c$ has acceptable physical properties, one would conclude on the basis of fairly general renormalization group arguments that at least in this lattice model the gravitational

coupling can only *increase* with distance. In addition the rise of the gravitational coupling in the infrared region rules out the applicability of perturbation theory to the low energy domain, to the same extent that such an approach is deemed to be inapplicable to study the low-energy properties of asymptotically free gauge theories.

It is a remarkable property of quantum field theories that a wide variety of physical properties can be determined from a relatively small set of universal quantities. Namely the universal leading critical exponents, computed in the vicinity of some fixed point (or fixed line) of the renormalization group equations. In the lattice theory the presence of a fixed point or phase transition is often inferred from the appearance of non-analytic terms in invariant local averages, such as for example the average curvature defined as

$$< l^2 > \frac{< \int d^4 x \sqrt{g} \, R(x) >}{< \int d^4 x \sqrt{g} >} \; \equiv \; \mathcal{R}(k) \underset{k \to k_c}{\sim} -A_{\mathcal{R}} \, (k_c - k)^{4\nu - 1} \; . \tag{6}$$

From such averages one can infer the value for ν, the correlation length exponent,

$$\xi(k) \underset{k \to k_c}{\sim} \; A_\xi \, (k_c - k)^{-\nu} \; . \tag{7}$$

In terms of the physical correlation length ξ one has

$$\mathcal{R}(\xi) \underset{k \to k_c}{\sim} \; \xi^{1/\nu - 4} \; . \tag{8}$$

Correct dimensions here can be restored by supplying appropriate powers of the ultraviolet cutoff, the Planck length $l_P = \sqrt{G}$. The fundamental critical exponent ν is related to the derivative of the beta function for G

$$\beta(G) \; \equiv \; \frac{\partial G}{\partial \log \mu} \tag{9}$$

in the vicinity of the ultraviolet fixed point at G_c,

$$\beta'(G_c) \; = \; -1/\nu \; . \tag{10}$$

Integrating close to the non-trivial fixed point one obtains for $G > G_c$

$$m = \xi^{-1} = \Lambda \exp\left(-\int^G \frac{dG'}{\beta(G')} \right) \underset{G \to G_c}{\sim} \Lambda \, |\, k_c - k\,|^{-1/\beta'(G_c)} \quad , \tag{11}$$

where m is an integration constant identified with the inverse correlation length, and $\Lambda = l_P^{-1}$ the ultraviolet cutoff.

This physical correlation length ξ determines the long-distance decay of the connected invariant two-point correlations at fixed geodesic distance d. For example for the curvature correlation one has, for distances much larger compared to the correlation length ξ,

$$< \sqrt{g} \, R(x) \, \sqrt{g} \, R(y) \, \delta(|x - y| - d) >_c \underset{d \gg \xi}{\sim} \; d^{-\sigma} \, e^{-d/\xi} \; , \tag{12}$$

while for short distances one expects a power law decay

$$< \sqrt{g} \, R(x) \, \sqrt{g} \, R(y) \, \delta(|x - y| - d) >_c \underset{d \ll \xi}{\sim} \; \frac{1}{d^{\, 2 \, (4 - 1/\nu)}} \; . \tag{13}$$

Method	k_c	ν
$\mathcal{R}$ vs. k	0.0630(11)	0.330(6)
$\mathcal{R}^3$ vs. k	0.0639(10)	-
$\chi_\mathcal{R}$ vs. k	0.0636(30)	0.317(38)
$\chi_\mathcal{R}^{3/2}$ vs. k	0.0641(17)	-
$\chi_\mathcal{R}/(\langle l^2 \rangle \mathcal{R})$ vs. k	0.0635(11)	0.339(9)
$\chi_\mathcal{R}$ vs. $\mathcal{R}$	-	0.328(6)
$\mathcal{R}$ FS scaling	-	0.333(2)
$\chi_\mathcal{R}$ FS scaling	-	0.318(10)

Table I: Summary table for the critical point k_c and the critical exponent ν, as obtained from lattices with up to 16^4 sites. The last three entries assume a critical point at $k_c = 0.0636$.

One further result is that according to the renormalization group the scale dependence of the effective Newton constant is given by

$$G(r) \;=\; G(0) \left[1 \,+\, c\,(r/\xi)^{1/\nu} \,+\, O((r/\xi)^{2/\nu}) \right] \;, \tag{14}$$

with c a calculable numerical constant. Here the scale ξ^{-1} plays a role very similar to the scaling violation parameter $\Lambda_{\overline{MS}}$ of QCD. It seems natural, although paradoxical at first, to associate ξ with some macroscopic cosmological length scale.

It should be clear from this brief discussion that the critical exponents provide a wealth of useful information about the continuum theory. In reality, the complexity of the lattice interactions and the practical need to sample many statistically independent field configurations contributing to the path integral leads to the requirement of powerful computational resources. The results discussed here [10] were obtained using a dedicated custom-built 20-GFlop 64-processor parallel supercomputer, described in detail elsewhere [11].

Table I [10] summarizes the results obtained for the critical point $k_c = 1/8\pi G_c$ and the critical exponent ν. From the numerical calculations one finds

$$k_c = 0.0636(11) \quad \nu = 0.335(9) \quad , \tag{15}$$

which clearly suggests $\nu = 1/3$ for pure quantum gravity.

4 Critical Exponents and Phenomenology

This section contains a brief discussion of some of the consequences that follow from the results presented in the previous section, and in particular the result $\nu = 1/3$.

First let us notice that the value $\nu = 1/3$ does not correspond to any known field theory or statistical mechanics model in four dimensions. Indeed for all scalar field theories (spin $s = 0$) in four dimensions it is known that $\nu = 1/2$, while for the compact Abelian U(1) gauge theory ($s = 1$) one has $\nu = 2/5$. The value $\nu = 1/3$ for $s = 2$ is then consistent with the simple interpolation $\nu^{-1} = 2 + s/2$.

One distinctive feature of the results is the appearance of a gravitational correlation length ξ. Naively one would expect, simply on the basis of dimensional

arguments, that the curvature scale should get determined by this correlation length

$$\mathcal{R} \underset{\mathcal{R} \to 0}{\sim} 1/\xi^2 \ , \tag{16}$$

but one cannot in general exclude the appearance of some non-trivial exponent. In the previous section arguments have been given in support of the value $\nu = 1/3$ for pure gravity. From the equation relating the average curvature to the gravitational correlation length one has

$$\mathcal{R}(\xi) \underset{k \to k_c}{\sim} \frac{1}{l_P^{2-d+1/\nu} \xi^{d-1/\nu}} \ . \tag{17}$$

Here the correct dimension for the average curvature $\mathcal{R}$ has been restored by supplying appropriate powers of the ultraviolet cutoff, the Planck length $l_P = \sqrt{G}$. For $\nu = 1/3$ in four dimensions one then obtains the remarkably simple result

$$\mathcal{R}(\xi) \underset{k \to k_c}{\sim} \frac{1}{l_P \, \xi} \ . \tag{18}$$

An equivalent form can be given in terms of the curvature scale H_0, defined through $R = -12 H_0^2$ which has dimensions of a mass squared. Then close to the critical point

$$H_0^2 \ = \ C_H \, \mu_P \, m \ , \tag{19}$$

where $\mu_P = 1/\sqrt{G}$ is the Planck mass, $m = 1/\xi$ is the inverse gravitational correlation length, and $C_H \approx 4.9$ a numerical constant of order one (the value for C is extracted from the known numerical values for $\mathcal{R}$ and m close to the critical point at k_c). It is amusing to note that this result is reminiscent of the pcac relation in pion physics.

One can raise the legitimate concern of how these results are changed by quantum fluctuations of matter fields. In the presence of matter fields coupled to gravity one expects the value for ν to change due to vacuum polarization loops containing these fields. But based on general arguments, one would expect fields whose masses are significantly above $m = 1/\xi$ to give negligible contributions to vacuum polarization loops, and thus leave the universal critical exponents which characterize the large distance behavior unaffected.

It seems natural to identify H_0 with either some (negative) average spatial curvature, or possibly with the Hubble constant determining the macroscopic expansion rate of the present universe [9]. In the Friedmann-Robertson-Walker model of standard cosmology on has for the Ricci scalar

$$R_{Ricci} \ = \ -6 \left\{ \left(\frac{\dot{R}}{R} \right)^2 \ + \ \frac{k}{R^2} \ + \ \frac{\ddot{R}}{R} \right\} \ , \tag{20}$$

where $R(t)$ is the FRW scale factor, and $k = 0, \pm 1$ for spatially flat, open or closed universes respectively. Today the Hubble constant is given by $H_0^2 = (\dot{R}/R)_{t_0}^2$, but it is eventually expected to show some slow variation in time. Its characteristic length scale today $c H_0^{-1} \approx 10^{28} cm$ is comparable to the extent of the visible universe. Under such circumstances one would expect the gravitational correlation ξ to be significantly larger than $c H_0^{-1}$.

A potential problem arises though in trying to establish a relationship between quantities which are truly constants (such as the ones appearing in Eq. (19)), and H_0 which most likely depends on time (the only exception being the steady state cosmological models, where H is truly a constant of nature; these models are not favored by present observations, including detailed features of the cosmic background radiation). In any case it is clear that some of these considerations are in fact quite general, to the extent that they rely on general principles of the renormalization group and are not tied to any particular value of ν, although the favored value $\nu = 1/3$ clearly has some aesthetic appeal.

One further observation can be made regarding the running of G in Eq. (14). Assuming the existence of an ultraviolet fixed point, the effective gravitational coupling is given for "short distances" $r \ll \xi$ by

$$G(r) \; = \; G(0) \left[1 \, + \, c \, (r/\xi)^3 \, + \, O((r/\xi)^6) \right] \; , \tag{21}$$

with c a calculable numerical constant of order one. The appearance of ξ in this equation, which is a very large quantity by Eq. (19), suggests that the leading scale-dependent correction which gradually increases the strength of the effective gravitational interaction as one goes to larger and larger length scales, should be exceedingly small. It also suggests that the deviations from classical general relativistic behavior for most physical quantities is in the end practically negligible.

It is only for distances comparable to or larger than ξ that the gravitational potential starts to weaken and fall off exponentially, with a range given by the gravitational correlation length ξ,

$$V(r) \; \underset{r \gg \xi}{\sim} \; - \, G(r) \, \frac{\mu_1 \mu_2 \; e^{-r/\xi}}{r} \; . \tag{22}$$

In many ways these results appear qualitatively consistent with the expected behavior of the tree-level graviton propagator in anti-de Sitter space. In the real world the range ξ must be of course very large; from the fact that super-clusters of galaxies apparently do form, one can set an observational lower limit $\xi > 10^{25} cm$.

It is unclear to what extent gravitational correlations can be measured directly. From the definition of the curvature correlation function in Eq. (13) one has for "short distances" $r \ll \xi$ and for the specific value $\nu = 1/3$ the remarkably simple result

$$< \sqrt{g} \, R(x) \, \sqrt{g} \, R(y) \, \delta(|x-y|-d) >_c \; \underset{d \ll \xi}{\sim} \; \frac{A}{d^2} \; , \tag{23}$$

with A a calculable numerical constant of order one. One should contrast this behavior with the semiclassical result attained close to two dimensions (and which incidentally coincides with the lowest order weak field expansion result), which gives instead for the power the value $2(d - 1/\nu) \sim 2(d - (d-2)) \sim 4$, as expected on the basis of naive dimensional arguments ($R \sim \partial^2 h$).

Next one can look at local curvature fluctuation. If one considers the curvature R averaged over a spherical volume $V_r = 4\pi r^3/3$,

$$\overline{\sqrt{g} \, R} \; = \; \frac{1}{V_r} \int_{V_r} d^3\vec{x} \; \sqrt{g(\vec{x}, t)} \, R(\vec{x}, t) \tag{24}$$

one can compute the corresponding variance in the curvature

$$[\delta(\sqrt{g}\,R)]^2 \;=\; \frac{1}{V_r^2} \int_{V_r} d^3\vec{x} \int_{V_r} d^3\vec{y} \; < \sqrt{g}\,R(\vec{x})\,\sqrt{g}\,R(\vec{y}) >_c \;=\; \frac{9A}{4\,r^2} \quad . \tag{25}$$

As a result the r.m.s. fluctuation of $\sqrt{g}R$ averaged over a spherical region of size r is given by

$$\delta(\sqrt{g}\,R) \;=\; \frac{3\sqrt{A}}{2}\,\frac{1}{r} \quad , \tag{26}$$

while the Fourier transform power spectrum at small $\vec{k}$ is given by

$$P_{\vec{k}} \;=\; |\,\sqrt{g}\,R_{\vec{k}}\,|^2 \;=\; \frac{4\pi^2 A}{2V}\,\frac{1}{k} \quad . \tag{27}$$

One can use Einstein's equations to relate the local curvature to the (primordial) mass density. From the Einstein field equations

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R \;=\; 8\pi T_{\mu\nu} \tag{28}$$

for a perfect fluid

$$T_{\mu\nu} \;=\; p g_{\mu\nu} + (p+\rho)u_\mu u_\nu \tag{29}$$

one obtains for the Ricci scalar, in the limit of negligible pressure,

$$R(x) \;\approx\; 8\pi G\,\rho(x) \quad . \tag{30}$$

As a result one expects for the density fluctuations a power law decay of the form

$$< \rho(x)\,\rho(y) >_c \;\underset{|x-y| \ll \xi}{\sim}\; \frac{1}{|x-y|^2} \quad . \tag{31}$$

Similar density correlations have been estimated from observational data by analyzing known galaxy number density distributions, giving a value for the exponent of about 1.77 ± 0.04 for distances in the $10kpc$ to $10Mpc$ range.

5 Outlook

Numerical simulation methods combined with modern renormalization group arguments can provide detailed information on non-perturbative aspects of a lattice model of quantum gravity. One then finds that the lattice model has two phases, only one of which is physically acceptable. In spite of the fact that the Euclidean theory becomes unstable as one approaches the critical point at k_c, it is still possible to determine by a straightforward analytic continuation the physical properties of the model in the vicinity of the true fixed point, defined as the point where a non-analiticity develops in the strong coupling branch of $Z_L(k)$. There scaling implies that the physical correlation ξ diverges.

If this prescription is followed, an estimate for the non-perturbative Callan-Symanzik beta function in the vicinity of the fixed point can be obtained, to leading order in the deviation of the bare coupling from its critical value. The resulting scale evolution for the gravitational constant is then quantitatively quite small, if one assumes that the scaling violation parameter is related to an average curvature

and its characteristic scale H_0. Its infrared growth, consistent with the general idea that gravitational vacuum polarization effects cannot exert any screening, suggests that low energy properties of quantum gravity are inaccessible by weak coupling perturbation theory: low energy quantum gravity is a strongly coupled theory. As pointed out in the discussion there are a number of attractive features to the pure gravity result $\nu = 1/3$, including a simple form for the curvature correlations at short distances.

It seems legitimate to ask the question whether the present lattice model for quantum gravity provides any insight into the problem of the cosmological constant. The answer is both yes and no. To the extent that a naive prediction of quantum gravity is that the curvature scale should be of the same order of the Planck length, $\mathcal{R} \sim 1/G$, the answer is definitely yes. Indeed it can be regarded as a non-trivial result of the lattice models for gravity that a region in coupling constant space can be found where space-time is stiff and the curvature can be made much smaller than $1/G$. In fact the evidence indicates that the average curvature $\mathcal{R}$ vanishes at the critical point k_c. And this is achieved with a bare cosmological constant λ which is of order one in units of the cutoff. Phrased differently, the dimensionless ratio between the renormalized and the bare cosmological constant becomes arbitrarily small towards the critical point.

At the same time the effective long distance cosmological constant is clearly non-vanishing and of order $1/\xi$ as a consequence of dimensional transmutation. The value zero is only obtained when ξ is exactly infinite, which happens only at the critical point k_c. Thus to make the effective cosmological constant small requires a fine tuning, in the sense that the bare coupling $k_c - k$ has to be small. But since the correlation length determines the corrections to the Newtonian potential (and in particular its eventual decrease for large enough distances), it would seem unnatural to have a short correlation length ξ: in such a world there would be no long-range gravitational forces, and separate space-time domains would have decoupled fluctuations. From this perspective, long range forces and a small cosmological constant go hand in hand. Quantum fluctuation effects show that hyperbolic space-times with small curvature radii cannot sustain long-range gravitational forces, at least in this lattice model.

6 Acknowledgments

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