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SCALAR FIELDS COUPLED TO FOUR-DIMENSIONAL LATTICE GRAVITY *

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ABSTRACT

I discuss some results we have obtained recently in a lattice model for quantized gravity coupled to scalar matter in four dimensions. We have looked at how the continuous phase transition separating the smooth from the rough phase of gravity is influenced by the presence of the scalar field. We find that close to the critical point, where the average curvature approaches zero, the effects of the scalar field are small and the coupling of matter to gravity seems to be weak. The nature of the phase diagram and the values for the critical exponents would suggest that gravitational interactions increase with distance.

Introduction

Of course any serious attempt at understanding the ground state properties of quantized gravity has to include at some stage the consideration of the effects of matter fields. While there are many choices for the matter fields and for their interactions, the simplest actions to deal with in the framework of a lattice model for gravity are the ones that represent one (or more) scalar fields [1]. In this talk I will briefly review these results. Regge's lattice model for gravity is the natural discretization for quantized gravity in four dimensions. At the classical level, it is completely equivalent to general relativity, and the correspondence is particularly transparent in the lattice weak field expansion, with the invariant edge lengths playing the role of infinitesimal geodesics in the continuum. Recent work based on Regge's simplicial formulation of gravity has shown, in pure gravity without matter, the appearance in four dimensions of a phase transition in the bare Newton's constant, separating a smooth phase with small negative average curvature from a rough phase with large positive curvature [2]. Here I will discuss a study we have performed to determine the nature and size of the effects that appear when a scalar field is coupled to gravity. As will be discussed below, our results seem to indicate that the 'vacuum polarization' effects due to one single scalar field of small mass are rather small for the observables we have investigated, when compared to the dominant pure gravitational contribution.

Action and Measure

For the gravitational field the following lattice action is used [3]

$$I_g[l] = \sum_{\text{hinges h}} \Big[\lambda V_h - k A_h \delta_h + a \frac{A_h^2 \delta_h^2}{V_h} \Big], \tag{1}$$

where V_h is the volume per hinge (which is represented by a triangle in four dimensions), A_h is the area of the hinge and δ_h the corresponding deficit angle, proportional to the curvature at h. All these quantities can be evaluated in terms of the lattice edge lengths l_{ij}, which define the lattice geometry for a fixed incidence matrix. The underlying lattice is chosen to be hypercubic, with a natural simplicial subdivision to make it rigid. In the classical continuum limit the above action is then equivalent to

$$I_g[g] = \int d^4x \sqrt{g} \Big[\lambda - \tfrac{1}{2} k R + \tfrac{1}{4} a R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \cdots \Big], \tag{2}$$

with a cosmological constant term (proportional to λ), the Einstein-Hilbert term ($k = 1/8\pi G$), and a higher derivative term proportional to a. For an appropriate choice of bare couplings, the above lattice action is bounded below for a

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regular lattice, even for $a = 0$, due to the presence of a lattice momentum cutoff, which cuts off the conformal mode fluctuations at high momenta. For non-singular measures and in the presence of the λ-term such a regular lattice can be shown to arise naturally. The higher derivative terms can be set to zero ($a = 0$), but they nevertheless seem to be necessary for reaching the lattice continuum limit, and are in any case generated by radiative corrections already in weak coupling perturbation theory.

The scalar field ϕ_i is then defined at the vertices of the simplices. One adds to the pure gravitational action the contribution

$$I_\phi[l,\phi] = \tfrac{1}{2} \sum_{<ij>} V_{ij} \left(\frac{\phi_i - \phi_j}{l_{ij}} \right)^2 + \tfrac{1}{2} \sum_i V_i \, (m^2 + \xi R_i) \phi_i^2 . \tag{3}$$

The term containing the discrete analog of the scalar curvature involves

$$V_i R_i \equiv \sum_{h \supset i} \delta_h A_h \sim \sqrt{g} R, \tag{4}$$

and in the expression for the scalar action, V_{ij} is the volume associated with the edge l_{ij}, while V_i is associated with the site i. Then the above scalar lattice action then corresponds to the continuum expression

$$I_\phi[g,\phi] = \tfrac{1}{2} \int \sqrt{g} \, [\, g^{\mu\nu} \, \partial_\mu \phi \, \partial_\nu \phi + (m^2 + \xi R) \phi^2] + \cdots . \tag{5}$$

The dimensionless coupling ξ is arbitrary; in our work we have only considered the case ($\xi = 0$) (minimal coupling).

The measure contains an integration over the scalar fields, and an integration over the edge lengths. For the edge lengths we write the lattice measure as [3]

$$\int d\mu_\epsilon[l] = \prod_{\text{edges ij}} \int_0^\infty \, V_{ij}^{2\sigma} \, dl_{ij}^2 \, F[l], \tag{6}$$

where V_{ij} is the 'volume per edge', $F[l]$ is a function of the edge lengths which enforces the higher-dimensional analogs of the triangle inequalities, and $\sigma = 0$ for the lattice analog of the DeWitt measure for pure gravity. In the presence of an n_f-component scalar field, the power σ needs to be changed, and on the lattice one has $\sigma = n_f/30$, since with our discretization of spacetime based on the simplicial subdivision of hypercubes there are a total of $2(2^d - 1) = 30$ edges emanating from each lattice vertex. Note that *no* cutoff is imposed on small or large edge lengths, if a non-singular measure such as dl^2 is used. This fact is essential for the recovery of diffeomorphism invariance close to the critical point, where on large lattices a few rather long edges, as well as some rather short ones, start to appear.

Observables

When we consider gravity coupled to a scalar field, we can distinguish two types of observables, those involving the metric field (the edge lengths) only, and those involving also the scalar field. The average curvature

$$\mathcal{R}(\lambda, k, a) \; \sim \; \frac{< \int \sqrt{g} \, R >}{< \int \sqrt{g} >}, \tag{7}$$

belongs to the first class, and its lattice analog is defined as

$$\mathcal{R}(\lambda, k, a) \; = < l^2 > \frac{< 2 \sum_h \delta_h A_h >}{< \sum_h V_h >}. \tag{8}$$

In a similar way one can define the curvature fluctuation. It is related to the (connected) scalar curvature correlator at zero momentum

$$\chi_{\mathcal{R}} \sim \frac{\int d^4x \int d^4y < \sqrt{g} R(x) \sqrt{g} R(y) >_c}{< \int d^4x \sqrt{g} >}. \tag{9}$$

A divergence in the curvature fluctuation is then indicative of long range correlations (a massless graviton here). Close to the critical point one expects for large separations a power law decay in the geodesic distance,

$$< \sqrt{g} R(x) \sqrt{g} R(y) > \underset{|x-y| \to \infty}{\sim} \frac{1}{|x-y|^{2n}}. \tag{10}$$

In quantum gravity it is of interest to determine the value of the low energy, renormalized coupling constants, and in particular the effective cosmological constant λ_{eff} and the effective Newton's constant G_{eff}. Equivalently, one would like to be able to determine the long distance limiting value of a dimensionless ratio such as $\lambda_{eff} G_{eff}^2$, and its dependence on the linear size of the system $L = V^{1/4}$. We have argued that the vacuum expectation value of the scalar curvature can be used as a definition of the effective, long distance cosmological constant

$$\mathcal{R} \sim (\lambda G)_{eff} \,. \tag{11}$$

Indeed in the pure gravity case there is evidence that the curvature vanishes at a critical point in G, and for $G > G_c$ one finds results which are consistent with a singularity of the type

$$\mathcal{R} \underset{G \to G_c}{\sim} -A_{\mathcal{R}} \, (G - G_c)^{\delta}, \tag{12}$$

where δ is a universal critical exponent [2]. If this is true, then $(\lambda G)_{eff} \to 0$ in lattice units at the critical point. While the location of the critical point G_c and the amplitude in general depend on the higher derivative coupling a and other non-universal parameters, the exponent is expected to be universal. We have estimated its value at about 0.6.

One immediate consequence of this result is that in the smooth phase with $G > G_c$ the gravitational coupling constant G must increase with distance (anti-screening), at least for rather short distances. Introducing an arbitrary momentum scale μ, one has close to the ultraviolet fixed point the following short-distance behavior for Newton's constant

$$G(\mu) - G_c \, = \, [G(\Lambda) - G_c] \left(\frac{\Lambda}{\mu} \right)^{1/\nu} , \tag{13}$$

with Λ the ultraviolet cutoff; the exponents δ and ν are calculable and are related to each other via the scaling relation $\nu = (1+\delta)/4 \approx 0.4$. Note that the opposite behavior (screening) would be true in the phase with $G < G_c$, but such a phase is known not to be stable and leads to no lattice continuum limit.

Switching on the Scalar Field

In order to explore the ground state of four-dimensional simplicial gravity coupled to matter beyond perturbation theory one has to resort to numerical methods, where the edge lengths and scalars are updated by a Monte Carlo method. In our work we considered lattices with 4^4 to 16^4 sites. As far as the properties of the critical point are concerned, one still finds an apparently continuous phase transition between the smooth and the rough phase, but with $A_{\mathcal{R}}, G_c, \delta$ that will now depend on n_f. If one adopts the same procedure as for pure gravity, and fits the average curvature for small scalar mass to an algebraic singularity, one finds (for $a = 0.005$ and $m = 0.5$) $\delta = 0.61(6)$, to be compared with the pure gravity estimate $\delta = 0.63(3)$ [2]. It appears therefore that, within errors, switching on the scalar fields leaves the exponents almost unchanged. For small non-integer n_f we can write for the amplitude, critical value of k and the exponent in powers of the number of flavors n_f,

$$\begin{aligned} A_{\mathcal{R}} \quad &= A_0 + n_f A_1 + O(n_f^2) \\ G_c \quad &= G_0 + n_f G_1 + O(n_f^2) \\ \delta \quad &= \delta_0 + n_f \delta_1 + O(n_f^2). \end{aligned} \tag{14}$$

Our results seem to imply that the corrections due to the scalar field are quite small, and that the coefficients of the n_f terms must be rather small. Since G_c and δ are almost unchanged, one can estimate A_1 in the following way. One notes that for small n_f the difference between the average curvature in the presence of the scalar field and in pure gravity determines the ratio of curvature amplitudes A_1/A_0

$$\frac{\mathcal{R}_{matter}}{\mathcal{R}_{gravity}} = \frac{\mathcal{R}_{gravity+matter} - \mathcal{R}_{gravity}}{\mathcal{R}_{gravity}} \underset{G \to G_c}{\sim} \frac{A_1}{A_0}. \tag{15}$$

Since the difference in the numerator is quite small, a very accurate measurement of the average curvature in both cases is required. One finds $A_1/A_0 \approx 0.053/3.79 = 0.014$ for $a = 0.005$. A possible explanation for the smallness of this ratio can be found in the Nielsen-Hughes formula, where the strength of the relative contributions (here gravity versus a scalar) is determined by the particle's relative spin.

Since the effects of the scalar fields are quite small, one can discuss the renormalization properties of the gravitational coupling constants without distinguishing the two cases explicitly. Let us address here briefly the renormalization properties of the couplings G and λ. Consider a universe of finite linear extent L, and set the graviton mass equal to the inverse of this size (since essentially the associated correlation length ξ saturates at the system size, $\xi \sim (G - G_c)^{-\nu} \sim L$).

From Eq. (13) one has then the following size dependence for the dimensionful Newton's constant, valid for 'short' distances $1/\mu \ll L$,

$$G_{eff}(\mu) \underset{L,\ 1/\mu \gg l_0}{\sim} l_0^2 G_c + l_0^2 \left(\frac{1}{\mu L} \right)^{1/\nu} \tag{16}$$

(with $1/\nu \approx 2.46$), while from Eqs. (11) and (12) one obtains

$$\lambda_{eff}(\mu) \underset{L,\ 1/\mu \gg l_0}{\sim} l_0^{-4} \, (\mu l_0)^{4-1/\nu} \left[G_c + \left(\frac{1}{\mu L} \right)^{1/\nu} \right]^{-1} . \tag{17}$$

Here again l_0 is of the order of the average lattice spacing, and we have restored the correct dimensions for G_{eff} and λ_{eff}. For the dimensionless ratio $(G^2 \lambda)_{eff}$ one then finds that it can be made very small, provided the linear size L is large enough.

Conclusions

Our work up to now suggests that the feedback of the scalar fields on the geometry is quite small on purely gravitational quantities, such as the average curvature. It appears therefore that the approximation in which matter internal loops are neglected (quenched approximation) could be considered a reasonable starting point, and that quantities such as the critical exponents should not be too far off in this case. To the extent that the coupling between the scalar and metric degrees of freedom is weak close to the critical point, we have argued that gravity is indeed weak. Our results for the exponents and the overall phase diagram seem to suggest that the gravitational coupling exhibits an infrared growth away from the fixed point, of the type $G(\mu) \sim \mu^{-1/\nu}$, while for the cosmological constant we have found a decrease in the infrared, $\Lambda(\mu) \sim \mu^{4-1/\nu}$, with an exponent ν given approximately by $\nu \approx 0.4$, and only weakly dependent on the matter content.

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