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AENEAS

A Custom-built Parallel Supercomputer for Quantum Gravity

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ABSTRACT

Accurate Quantum Gravity calculations, based on the simplicial lattice formulation, are computationally very demanding and require vast amounts of computer resources. A custom-made 64-node parallel supercomputer capable of performing up to 2×10^{10} floating point operations per second has been assembled entirely out of commodity components, and has been operational for the last ten months. It will allow the numerical computation of a variety of quantities of physical interest in quantum gravity and related field theories, including the estimate of the critical exponents in the vicinity of the ultraviolet fixed point to an accuracy of a few percent.

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1 Physics Goals

One of the outstanding problems in theoretical physics is the determination of the quantum-mechanical properties of Einstein's relativistic theory of Gravitation. Approaches based on linearized perturbation methods have not been successful so far, as the underlying theory is known not to be perturbatively renormalizable. Due to the complexity of even such approximate calculations, a fundamental coupling of the theory, the bare cosmological constant term is usually set to zero, thus further reducing the potential physical relevance of the results. Furthermore, gravitational fields are themselves the source for gravitation, which leads to the problem of a highly non-linear theory, where any sort of perturbative results is possibly of doubtful validity, especially in the quantum domain, where strong fluctuations in the gravitational fields appear at short distances.

The above-described situation bears some resemblance to the theory of strong interactions, Quantum Chromodynamics. Non-linear effects are known here to play an important role, and end up restricting the validity of perturbative calculations to the high energy, short distance regime where the effective coupling can be considered weak. For low energy properties Wilson's discrete lattice formulation, combined with computer simulations, has provided so far the only convincing evidence for quark confinement and chiral symmetry breaking, two phenomena which are completely invisible to any order in the weak coupling, perturbative expansion.

A discrete lattice formulation can be applied to the problem of quantizing gravity as well. Instead of continuous fields, one deals with gravitational fields which live only on discrete space-time points and interact locally with each other. In Regge's simplicial formulation of gravity [1] one approximates the functional integration over continuous metrics by a discretized sum over piecewise linear simplicial geometries [2, 3]. In such a model the role of the continuum metric is played by the edge lengths of the simplices, while curvature is described by a set of deficit angles, which can be computed via standard formulae as functions of the given edge lengths. The simplicial lattice formulation of gravity is locally gauge invariant [4], and is known to contain perturbative gravitons in the lattice weak field expansion, making it an attractive lattice regularization of the continuum theory.

The starting point for a non-perturbative study of quantum gravity is a suitable definition of the discrete Feynman path integral. In the simplicial lattice approach one starts from the discretized Euclidean path integral for pure gravity, with the squared edge lengths as fundamental variables,

$$Z_L \;=\; \int_0^\infty \prod_s \; (V_d(s))^\sigma \; \prod_{ij} dl_{ij}^2 \; \Theta[l_{ij}^2] \; \exp\left\{ - \sum_h \left(\lambda\, V_h - k\, \delta_h A_h + a\, \frac{\delta_h^2 A_h^2}{V_h} + \cdots \right) \right\} \quad . \tag{1.1}$$

The above expression represents an elegant discretization of the continuum Euclidean path integral for pure quantum gravity

$$Z_C = \int \prod_x \left(\sqrt{g(x)}\right)^\sigma \prod_{\mu \geq \nu} dg_{\mu\nu}(x) \, \exp\left\{ - \int d^4 x \, \sqrt{g} \left(\lambda - \frac{k}{2} R + \frac{a}{4} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \cdots \right) \right\} \quad , \tag{1.2}$$

with $k^{-1} = 8\pi G$, and G Newton's constant, and reduces to it for smooth enough field configurations. In the discrete case the integration over metrics is replaced by integrals over the elementary lattice degrees of freedom, the squared edge lengths. The above discrete gravitational measure is then the lattice analog of the DeWitt continuum functional measure [5, 6]. The δA term in the lattice action is the well-known Regge term, and reduces to the Einstein-Hilbert action in the lattice continuum limit. A cosmological constant term is needed for convergence of the path integral, while the curvature squared term allows one to control the fluctuations in the curvature. In practice, and for phenomenological reasons, one is only interested in the limit when the higher derivative terms are small, $a \to 0$. In this limit the theory depends, in the absence of matter and after a suitable rescaling of the metric, only on one bare parameter, the dimensionless coupling k^2/λ.

The discretized theory contains only a finite set of variables, once a set of suitable boundary conditions are imposed, such as open or periodic. In the end the original continuum theory of gravity is to be recovered as the space-time volume is made large and the fundamental lattice spacing of the discrete theory is sent to zero, possibly without having to rely, at least in principle, on any further approximation to the original continuum theory.

Quantum fluctuations in the underlying geometry are represented in the discrete theory by fluctuations in the edge lengths, which can be modeled by a well-defined Monte-Carlo stochastic process. In analogy with other field theory models studied by computer, all calculations so far have been performed in the Euclidean, imaginary time framework, which is the only formulation amenable to a controlled numerical study, at least for the foreseeable future. The Monte-Carlo method, based on the concept of importance sampling, is well suited for evaluating the discrete path integral for gravity and for computing the required averages and correlation functions. By a careful analysis of the lattice results, the critical exponents can be extracted, and the scaling properties of invariant correlation functions determined from first principles.

Studies on small lattices suggest a rich scenario for the ground state of quantum gravity [7]. The present evidence suggests that simplicial quantum gravity in four dimensions exhibits a phase transition (in G) between two phases: a strong coupling phase, in which the geometry is smooth at large scales and quantum fluctuations in the gravitational field are bounded, and a weak coupling phase, in which the geometry is degenerate and space-time collapses into a lower-dimensional manifold. Only the smooth, small negative curvature AdS phase appears to be physically acceptable. The existence of a phase transition at finite coupling G implies non-trivial, calculable non-perturbative scaling properties for the correlations and coupling constants of the theory, and in particular Newton's constant. Its presence is usually inferred from the presence of non-analytic terms in invariant averages, such as for example the average curvature

$$< l^2 > \frac{< \int d^4x \sqrt{g}\, R(x) >}{< \int d^4x \sqrt{g} >} \;\equiv\; \mathcal{R}(k) \underset{k \to k_c}{\sim} -A_{\mathcal{R}} \; (k_c - k)^{4\nu - 1} \quad , \tag{1.3}$$

From such averages one can determine the value for ν, the correlation length exponent,

$$\xi(k) \underset{k \to k_c}{\sim} A_\xi \; (k_c - k)^{-\nu} \quad . \tag{1.4}$$

An equivalent result, relating the quantum expectation value of the curvature to the physical correlation length ξ , is

$$\mathcal{R}(\xi) \underset{k \to k_c}{\sim} \xi^{1/\nu - 4} \quad . \tag{1.5}$$

Matching of dimensionalities in these equations is restored by supplying appropriate powers of the Planck length $\sqrt{G}$. The exponent ν is known to be related to the derivative of the beta function for G in the vicinity of the ultraviolet fixed point,

$$\beta'(G_c) \;=\; -1/\nu \quad . \tag{1.6}$$

In addition, the correlation length ξ itself determines the long-distance decay of the connected, invariant correlations at fixed geodesic distance d. Thus for the curvature correlation one has at large distances

$$< \sqrt{g}\; R(x) \; \sqrt{g}\; R(y) \; \delta(|x - y| - d) >_c \underset{d \,\gg\, \xi}{\sim} d^{-\sigma} \; e^{-d/\xi} \quad , \tag{1.7}$$

while for shorter distances one expects a slower power law decay

$$< \sqrt{g}\; R(x) \; \sqrt{g}\; R(y) \; \delta(|x - y| - d) >_c \underset{d \,\ll\, \xi}{\sim} \frac{1}{d^{\,2(4 - 1/\nu)}} \quad . \tag{1.8}$$

The scale dependence of the effective Newton constant is given by

$$G(r) \;=\; G(0) \left[1 \,+\, c \,(r/\xi)^{1/\nu} \,+\, O((r/\xi)^{2/\nu}) \right] \quad , \tag{1.9}$$

with c a calculable numerical constant. In this last expression the momentum scale ξ^{-1} plays a role somewhat similar to the scaling violation parameter $\Lambda_{\overline{MS}}$ of QCD. Hopefully it should be clear, even from this brief discussion, that the critical exponents by themselves already provide a significant amount of useful information about the continuum theory.

In reality, the complexity of the interaction and the need to sample many statistically independent field configurations in the path integral, which is necessary for correctly incorporating into the model the effects of quantum-mechanical fluctuations, leads to the requirement of powerful computational resources.

2 The Machine

In this section the main physical characteristics of the machine will be described. Although the Aeneas computer was built with quantum field theory applications in mind, it is in fact designed as a general purpose supercomputer and therefore suitable for a wide range of computationally intense applications. Its layout is similar to other parallel distributed memory architectures, such as the IBM SP2, the Intel Paragon or the Cray T3E, although the switching network is not custom made and therefore not as fast. To keep the costs under control and achieve a high performance over cost ratio, it has been assembled out of relatively easy available hardware and software components. The essential features of the machine are

- Use of commodity microprocessors and data buses, allowing the use of reliable and cost-effective off-the-shelf technology,
- Fast Ethernet (100 MegaBits/sec) connectivity between processor nodes,
- Reliable, well tested public domain software, based on Linux for the operating system, on public domain Gnu tools and compilers, and on MPIch and PVM for inter-processor communication.

Before AENEAS was assembled, the above combination of commodity hardware and public domain software tools had been tested on similar machines based on Pentium Pro processors built at NASA GSFC, CalTech and LANL. The present design of Aeneas differs from these earlier prototypes in that it uses faster 300 MHz PII processors, newer design motherboards and larger, faster Ethernet switches.

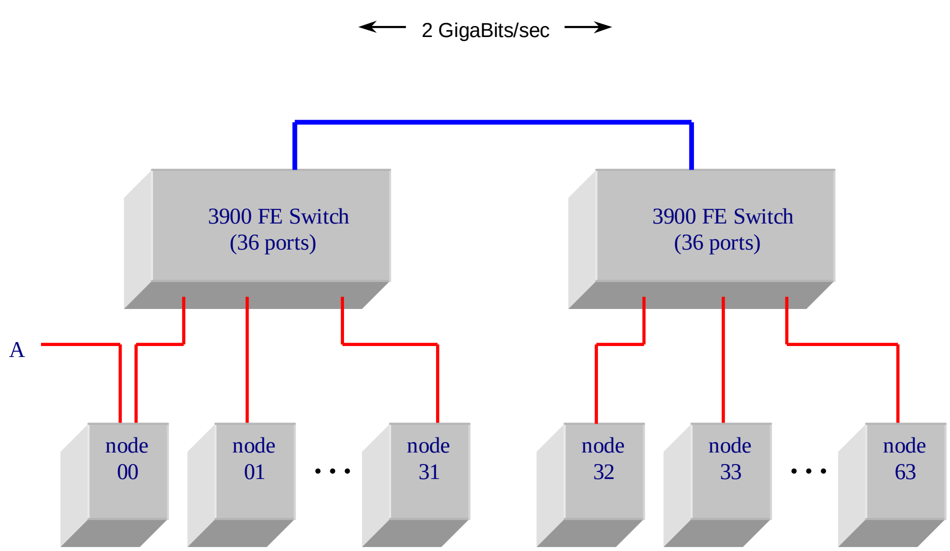
In more detail, the hardware consists of 65 nodes, each based on

- an Intel Pentium II processor with 512k cache running at 300 MHz
- a PCI motherboard based on the Intel 440LX chipset
- 128 MegaBytes of 168-pin 10-ns SDRAM
- one 3.2 GBytes Quantum ST E-IDE disk drive
- one D-Link Fast Ethernet adapter with the DEC Tulip 21142 chipset
- a floppy drive and an S3 Virge video card for maintenance access

Multiple ethernet cards are installed in the front-end node in order to provide external access to the cluster, isolating at the same time the internal network traffic from the outside office network. Alternatives to Fast Ethernet, such as Myrnet, offer slightly higher performance, but at a much higher price per node.

Given that a major bottleneck in the design is the speed of the PCI bus (66 MHz), it seemed unwise to afford the extra cost without reaping significant performance improvement benefits. Faster processors (such as the DEC alpha) would improve single node performance (especially on long vector operations), but would still face the networking bottlenecks for communication-intensive parallel applications. Fast (pio-4) IDE disks where chosen because of their lower costs in comparison to SCSI disks, which offer higher performance but at a significantly higher price. Reliability is comparable for both types of disks, as the disk and heads employ similar technology. Several nodes, including the front end, have additional larger 6.4 GigaBytes IDE disks, to ease the movement of large data sets in and out of the parallel machine. Three spare nodes, as well as additional cloned spare disks, ensure brief downtimes (of about 20 minutes or less) in case of a hardware failure.

Connectivity between the processor nodes is provided by two 36-port 3-Com Fast Ethernet (100 MegaBits/sec) crossbar switches, which are connected to the processor nodes via standard cat5 twisted pair cables. At full duplex, any two nodes can exchange data with each other over the switch at up to 25 MegaBytes/sec, the peak hardware bandwidth. The Fast Ethernet crossbar switches themselves have a backplane switching capability of 6.6 GigaBits/sec, and are connected to each other via multiple trunked full duplex Gigabit Ethernet SC multi-mode fiber connections (using the available expansion modules), giving a peak hardware bandwidth between the two switches of 500 MegaBytes/sec.



Network topology of the Aeneas parallel supercomputer.

Power consumption is about 50 Watts/node. As a significant amount of heat is dissipated by the 64 nodes, the machine is housed in a machine room with a large chilled water cooling unit, ensuring a constant 72^0F temperature throughout the room. The total cost of the hardware, including tax, is about ninetytwothousand dollars at September 1998 market prices.

The software installed on all nodes includes

- the RedHat Linux distribution, with updated Ethernet drivers
- Gnu g77, C, C++ compilers, and Absoft's f77 and f90 compilers
- MPI and PVM for message passing between processor nodes
- additional libraries such as CernLib and Lapack
- a queuing system based on DQS

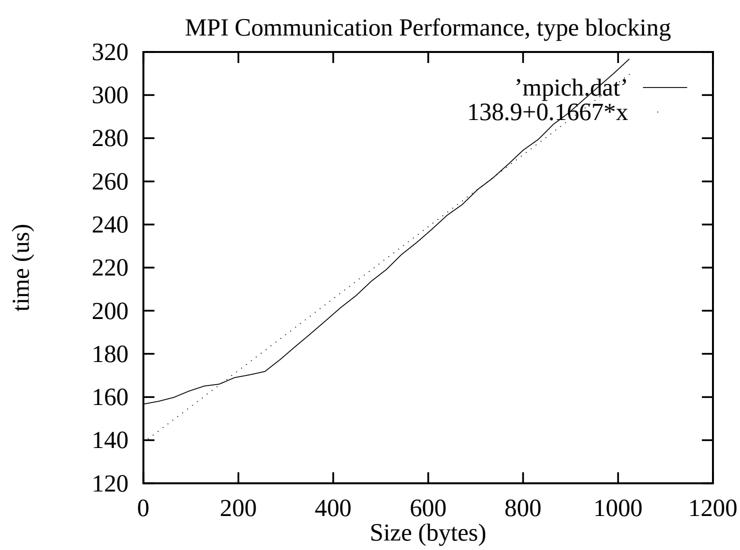
The RedHat Linux distribution was chosen because of its ease of installation and maintenance via rpm technology. The software on all disks is cloned, providing for the same user interface, compilers, libraries etc. on all nodes. The latest releases of Linux (RH4.2-5.1) are very stable, and the present cluster has registered uptimes of the order of months on all nodes, which are clock synchronized via an xntp time server. Both g77 and Absoft's f77 and f90 compilers work well with the MPI distribution from Argonne National Lab (MPIch), with the Absoft f77 and f90 compilers typically generating faster code than g77.

3 Performance

In this section various performance aspects of the Aeneas machine on a variety of codes will be discussed.

Single node code performance depends on a variety of factors, including array sizes, cache size compatibility, quality of coding, version of libraries used and quality of compiler optimization. It is not unusual for typical Fortran code to achieve floating point performance between 60 and 80 MegaFlops on one node of Aeneas. For long vector operations the MegaFlop rate rises to anywhere between 90 and 250, with judicious use of unrolling and an appropriate use of compiler switches. Typically the Pentium II at 300 MHz is about 50% faster than the 200 MHz Pentium Pro (i.e. close to the ratio of clock speeds), whit Absoft's f77 compiler typically generating faster code (by as much as 32%) than g77. The standard Linpack benchmark on a 100 by 100 matrix multiply gives 85 MegaFlops in single precision, versus 66 MegaFlops in double precision, a 22% degradation in speed. In general, the Pentium II's performance versus the Mips R10000, the IBM P2SC and the Dec Alpha processors is quite good for scalar-dominated code, but the Intel processor is somewhat out-classed by code which can take advantage of the multiple arithmetic units available on the other three processors.

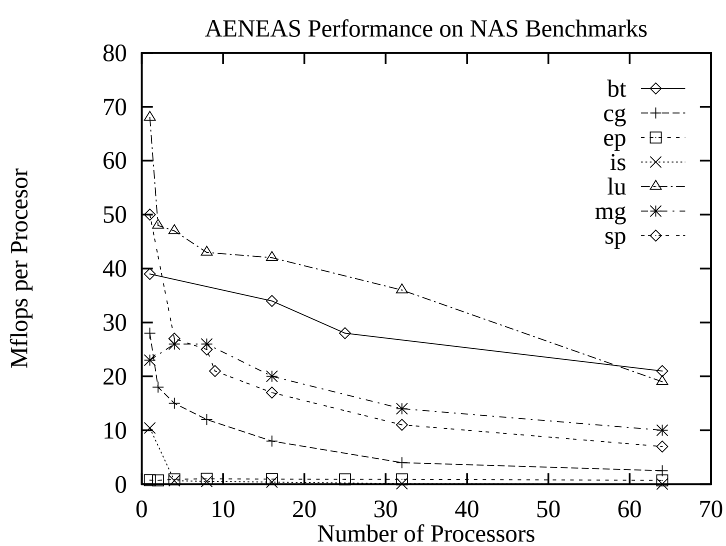
Most parallel applications developed so far embed MPI (Message Passing Interface, version 1.1, from Argonne National Lab) library calls in f77 or f90 programs, although there is some limited PVM (Parallel Virtual Machine, from Oak Ridge National Lab) parallel programming experience. In general MPI is preferred because of lower latencies and higher bandwidth, and therefore higher parallel performance. Most MPI codes have been ported from larger parallel machines, such as the Cray T3-E 1200 and the IBM SP2 (which use their own tuned MPI implementation). As the different MPI versions are virtually indistinguishable from one machine to another, very few code modifications are necessary. These mostly concern floating point precision issues (the ix86 processors are 32-bit chips) and calls to the real time clock. In general, MPI with either f77 or f90 appears to work flawlessly under Linux.



Communications performance benchmark, from MPIch distribution.

It seems useful to compare the communication parameters of Aeneas to other parallel supercomputers. The standard MPI distribution from ANL includes a set of test programs to evaluate the efficiency of parallel applications under MPI. Tools are included to estimate both bandwidth and latency in interprocessor communications. Between any two nodes one finds that the measured latency is about 150 $\mu secs$, while the measured communication bandwidth ranges between 0.38 MegaBytes/sec (for 64-byte packets) and 10.0 MegaBytes/sec (for 256k-byte packets). The latter transfer rate figure is close to the peak one-way hardware bandwidth limit of 12.5 MegaBytes/sec. The lesson here is that in general it pays to send one large packet between nodes, as opposed to several small ones. The above communications performance figures should be compared for example to an IBM SP2, for which the bandwidth is 120 MegaBytes/sec and the latency 52 μsec.

A number of standard parallel benchmarks have been run, with the intent of comparing Aeneas with similar parallel computers available commercially, and asses its viability as a machine for various problems involving quantum field theory simulations. For such a comparison, a particularly popular suite is the Nasa NAS (Numerical Aerodynamics Simulations) class B v. 2.3 set of parallel benchmarks, most of which consist of various Navier-Stokes equation solver fragments, used mostly in Computational Fluid Dynamics (CDF) applications. Some of them involve a fair amount of linear algebra distributed across the processors. The NAS benchmarks clearly show the effectiveness of Aeneas as a general-purpose parallel supercomputer, compared to larger and much more costly machines (a 64-node Cray Origin 2000, for example, has a list price of about \$1.9M). Shown above, as an example, is a performance comparison for the LU matrix decomposition benchmark with the latest Cray Origin 2000 (one should perhaps add that none of the quoted NAS benchmarks have been tuned for the Aeneas architecture). When a large number of nodes is used, the performance degradation in the NAS benchmarks becomes more significant due to the limitations in the network speed (communications overhead for N nodes generally grows like $N \log_2 N$). Not unexpectedly, Aenas seems to be doing well performance-wise when there is not an excessive amount of communication between the nodes.



Scaling of NAS parallel benchmarks with processor nodes.

As a final example, the performance of the simplicial quantum gravity code is shown in comparison to

	4 nodes	8 nodes	16 nodes	32 nodes
Aeneas (300 MHz)	187	346	675	1142
Cray Origin 2000 (195 MHz)	367	696	1383	2991

Nasa NAS Class B v2.3 parallel benchmark, LU decomposition. Rates in MegaFlops.

	1 node	16 nodes	32 nodes
TMC CM5	36340 (**2**)	2271 (**2**)	1136 (**2**)
IBM SP2 (160 MHz)	873 (**95**)	69 (**75**)	36 (**73**)
Aeneas (300 MHz)	1090 (**77**)	83 (**63**)	48 (**55**)

Performance of Simplicial Gravity code. Wall clock times in seconds, MegaFlops per node in parenthesis.

other similar parallel machines. As will become clear below, the gravity application is well suited for a parallel architecture, and was in fact originally developed for the CM5, whose message passing calls were rather similar to MPI. Later it was adapted slightly to other architectures such as the IBM SP2 and the Cray T3E. The particular example shown in the table is for a lattice of 16^4 sites, although much larger lattices such as 32^4 are feasible. These types of simulation codes are well suited for Aeneas, as they involve relatively little communication between processors versus actual computation. Typically the update of one edge length requires of the order of $100k$ floating point operations, versus the exchange of only 8 Bytes of data between nodes after the update has been completed. Either 32 or 64 edges can be updated in parallel. On a lattice with 16^4 sites one can therefore typically generate up to about $100k$ metric configurations a month. The actual accuracy with which critical exponents and correlations can be determined depends of course on a number of factors, of which the lattice size and numerical accuracy are just two components.

The overall versatility of the machine is also brought out by the fact that a number of applications in disciplines outside of theoretical physics can use such an architecture for productive work. Additional parallel applications which have been deployed successfully on Aeneas include an atmospheric chemistry code for simulating the diffusion of pollutants in the atmosphere, and a fluid dynamics application involving turbine blade design. In addition, the machine has been used effectively as a high-powered cluster for high energy physics detector simulations and data analysis, as part of the Amanda and Super-Kamiokande experimental projects.

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