Numerical Estimates of Hadronic Masses in a Pure SU(3) Gauge Theory

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In lattice quantum chromodynamics, the hadronic mass spectrum is evaluated by computer simulations in the approximation where closed quark loops are neglected. Chiral symmetry is shown to be spontaneously broken and an estimate of the pion decay constant is given.

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In this Letter we present results of a computation of the mass spectrum of the lighter hadrons in the SU(3) lattice gauge theory in the approximation of neglecting internal quark loops. Although these effects will have to be taken into account in a full calculation, we shall see that reasonable results for the spectrum can be obtained within this framework. This approximation enforces the Zweig rule for all the flavors, becomes exact in the limit $N\to\infty$, and can easily be justified for the mass spectrum with phenomenological arguments. Some numerical simulations of two-dimensional lattice gauge models also suggest that it might be a reasonable simplification.[1] We have found some evidence for this to be true also in the present case. In this approximation nonet symmetry holds: Closed quark loops are crucial to remove the η-π degeneracy. Similar computations not including the baryons and with only one form of the fermionic action (the Kogut-Susskind action) for the group SU(2) will be published elsewhere.[2]

The mass spectrum of the lighter hadrons can be computed by studying the decay at infinity of the correlation functions of composite operators. The key formulas we use are

$$\langle\bar\psi(x)\psi(x)\bar\psi(0)\psi(0)\rangle = \int d_\mu[A]\,G(x,0;A)G(0,x;A),$$
$$\langle\bar\psi(x)\bar\psi(x)\bar\psi(x)\psi(0)\psi(0)\psi(0)\rangle \qquad (1)$$
$$= \int d_\mu[A]\,G(x,0;A)G(x,0;A)G(x,0;A),$$

where we have suppressed flavor, spinor, and color indices, and $G(x,0;A)$ is the inverse of $\not{D}+m$ in a background A_μ gauge-field configuration. D_μ is the covariant derivative and $d_\mu[A]$ is the probability distribution of pure gauge fields. These formulas hold for all operators that do not have the flavor quantum numbers of the vacuum. In the full theory with n_f fermion flavors and vacuum polarization effects included we would have

$$d_\mu[A] = e^{S_G}[\det(\not{D}+m)]^{n_f}\,dU_H,$$

where S_G is the Wilson action for lattice gauge fields given by

$$S_G = g_0^{-2}\sum_{\Box}\mathrm{Tr}(U_p + U_p^\dagger - 2),$$

and dU_H is the Haar measure for the group SU(3) for each link. The sum is over all elementary squares in the four-dimensional hypercubic lattice of spacing a, and U_p is a product of four SU(3) group elements around each square. In this Letter we will discuss results obtained by setting the determinant equal to 1 ($n_f=0$), which is equivalent to neglecting dynamic fermion loops. The fields A can be extracted by using a standard Monte Carlo simulation technique, while the inverse propagators are computed using iterative matrix inversion methods.[1]

When we implement these methods on the space-time lattice we have to make a choice regarding the fermionic action. In general we can write[3-7]

$$S(\bar\psi,\psi) = \sum_{n,\mu}\bar\psi_n[(\gamma_\mu - r)\psi_{n+\mu} - (\gamma_\mu + r)\psi_{n-\mu}] + M\sum_n\bar\psi_n\psi_n. \qquad (2)$$

If $r=0$ the theory is chirally invariant in the $M\to 0$ limit,[4-6] but unfortunately describes sixteen flavors instead of one. (These can be reduced to four by an appropriate canonical transformation, as discussed in Refs. 4 and 6.) If $r\neq 0$ only one flavor is obtained in the continuum limit, but chiral symmetry is lost on the lattice and can only be recovered in the continuum limit, as discussed in Ref. 3.

In this Letter we present results of a computation of the hadronic spectrum in the cases $r=0$ and $r=1$ on lattices $6^3\times 12$ and up to $6^3\times 10$, respectively, and compare the results.

We have used the standard Wilson action for the gauge fields[3] and generated the gauge-field distributions using a modified Monte Carlo method: We did ten trials for each gauge-field variable without changing the others. In this way the Monte Carlo method is very similar to the heat-bath method. We have limited ourselves to a study of the region of β between 5 and 6.2 (we use the notation $\beta=6/g_0^2$). The crossover between the weak- and strong-coupling regimes happens around $\beta\simeq 5$ and in the region we have studied one starts to see the exponential behavior in β of the string tension as predicted by asymptotic freedom. If for definiteness we assume $\sqrt{k}=7\times 10^{-3}\Lambda_0$ (Ref. 7) and $\sqrt{k}=420$ MeV from the ρ-f-g-h trajectory, our inverse lattice spacing ranges from 440 MeV at $\beta=5$ to 1590 MeV at $\beta=6.2$. At $\beta=5.6$ and 6.0 where we have higher statistics we obtain $a^{-1}=660$ MeV and $a^{-1}=1120$ MeV, respectively.

In this interval of β we have computed the expectation value of $\langle\bar{\psi}\psi\rangle$ at $r=0$ in the mass range $m=0.1$–0.3 on a 6^4 lattice using the Langevin algorithm of Ref. 8 and the iterative method, and then extrapolated to $m=0$ using a quadratic fit. We clearly see evidence for $\langle\bar{\psi}\psi\rangle\neq 0$ at $m=0$ for all values of β that we have explored. Chiral symmetry is spontaneously broken and the usual Goldstone theorem holds. The results extrapolated to $m=0$ are shown in Fig. 1. The line is a fit of the form

$$\langle\bar{\psi}\psi\rangle_{m=0}[(3\alpha_B)^{-4/33}R\sqrt{k}]^3 \quad (3)$$

following the prescription of the renormalization group. Here $\alpha_B=3/2\pi(\beta-2.75)$ and the scale parameter of quantum chromodynamics is defined as

$$\Lambda_{\text{mom}}=\frac{\pi}{a}\,\frac{8\pi^2}{33}\left[(\beta-2.75)\right]^{51/121}\times\exp\left[-\frac{4\pi^2}{33}(\beta-2.75)\right]. \quad (4)$$

The data suggest $R=0.90\pm 0.05$ in agreement with phenomenological estimates.[9]

At $r=0$ we have obtained some results for the mass spectrum of the lowest-lying states at $\beta=6.0$ by computing the propagator $G(x,0;A)$ for four different configurations. In the region of m between 0.3 and 0.1 in lattice units we can fit the data by

$$m_P^2=6m/a,\quad m_V^2=0.5/a^2+6m/a,\quad m_B=1.0/a+12m. \quad (5)$$

Here m_B^2 is the average mass of the lowest baryonic states [something between the nucleon and the Δ mass; we do not have enough statistics yet to see their mass difference in this formulation ($r=0$)]. This gives the rough estimates

$$m_\rho=800\pm 100\ \text{MeV},\quad m_B=1000\pm 100\ \text{MeV}. \quad (6)$$

The linear dependence of the pion mass on the bare quark mass is a consequence of the Goldstone theorem. If we use the partially conserved axial-vector current relation

$$m\langle\bar{\psi}\psi\rangle=f_\pi^2 m_\pi^2, \quad (7)$$

we get $f_\pi=95\pm 10$ MeV from the value of $\langle\bar{\psi}\psi\rangle$ at $\beta=6$ which is 0.043 ± 0.01 in lattice units. While the masses of the ρ meson and of the baryon do not appreciably change when varying β, we have noticed that f_π varies from the above value at $\beta=6$ to $f_\pi=140\pm 10$ MeV at $\beta=5.6$. This does not come entirely as a surprise since f_π is more or less the pion wave function at the origin and is therefore expected to be a more sensitive quantity than the masses.

We now come to a discussion of the results for $r=1$ (Wilson's fermions). By doing a fit to the

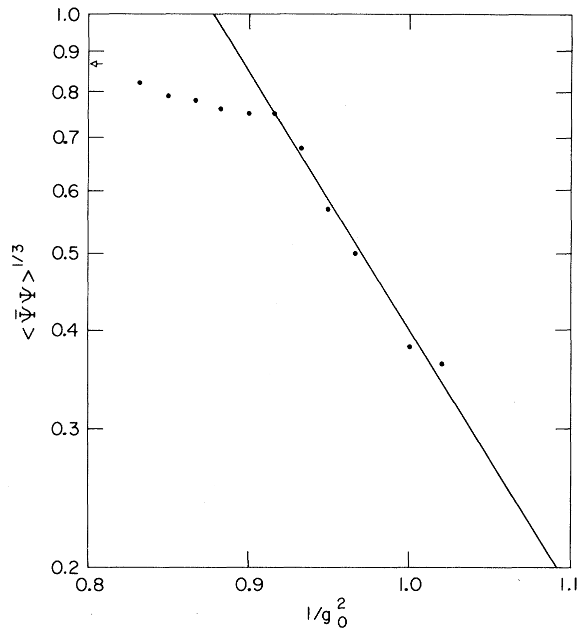

FIG. 1. The quantity $\langle\bar{\psi}\psi\rangle^{1/3}$ as a function of $1/g_0^2$. The line is a fit that gives $\langle\bar{\psi}\psi\rangle^{1/3}=1.5(3\alpha_B)^{-4/33}\Lambda_{\text{mom}}$.

data for m between 0.3 and 0.05 (see Fig. 2), where all the masses are between the low-energy cutoff at 590 MeV and the high-energy cutoff at 3520 MeV, one finds at $\beta = 6$

$$m_P^2 = 6.0m/a, \quad m_V^2 = 0.5/a^2 + 6m/a,$$
$$m_S^2 = 0.8/a^2 + 6m/a, \quad m_A^2 = 1.2/a^2 + 6m/a, \qquad (8)$$
$$m_N = 0.8/a + 7m, \quad m_\Delta = 1.1/a + 5m.$$

Here m is, in Wilson's notation, $(k_c - k)/2k_c^2$ with $k = 1/M$, and $k_c \simeq 0.156$ at $\beta = 6$. These results were obtained with fifty different configurations at several different values for the bare quark mass, and the error in the masses is of order 10%.

Using $a^{-1} = 1120$ MeV we get the following estimates (in MeV) (we use as input $m_\pi = 140$ MeV):

$$m_\rho = 800 \pm 100, \quad m_p = 950 \pm 100,$$
$$m_\delta = 1000 \pm 100, \quad m_\Delta = 1300 \pm 100, \qquad (9)$$
$$m_{A_1} = 1200 \pm 100, \quad f_\pi = 95 \pm 10.$$

Analogous estimates can be obtained for strange mesons and baryons. We could have alternatively chosen to fit both the π and the ρ masses to experiment, and this would have given us roughly the above value for the lattice spacing and $\sqrt{T} = 400 \pm 50$ MeV. The bare quark masses turn out to be

$$(3\alpha_B)^{-4/11}(m_u + m_d) = 8 \text{ MeV},$$
$$(3\alpha_B)^{-4/11}(m_u + m_s) = 100 \text{ MeV}, \qquad (10)$$

which then gives 3, 5, and 100 MeV for the u, d, and s invariant quark masses, in agreement with previous phenomenological estimates.[9] The results at $r = 0$ and $r = 1$ seem therefore to be compatible, within statistical errors. However, the $r = 1$ approach seems to be more promising for the study of the spectrum of hadrons since the separation of operators with different spins and parities can be done on a single site. Using the large number of configurations generated for the fermions, we have also estimated the lowest glueball mass by extracting the plaquette-plaquette correlations from finite-size effects[10] at $\beta = 6$, with the result

$$m_G = (3.6 \pm 0.5)\sqrt{k} = 1500 \pm 200 \text{ MeV}. \qquad (11)$$

We plan to improve further the accuracy of the spectrum calculation by increasing the statistics. Calculations of the spectrum on larger lattices are in progress in order to check the smallness of finite-size effects on the lattice. A comparison between the results on a $5^3 \times 8$ and a $6^3 \times 10$ lattice seems to show that these effects are small when the pion mass is not too close to the high- or low-energy cutoffs. A more detailed account of the spectrum calculation will be published elsewhere. We are also considering the possibility of introducing the effects of closed fermion loops.[1,8,11-13] One way of doing it would be to use chirally invariant fermions at $r = 0$ with one component per site for the fermionic loops and the noninvariant formulation at $r = 1$ for the computation of the propagators.

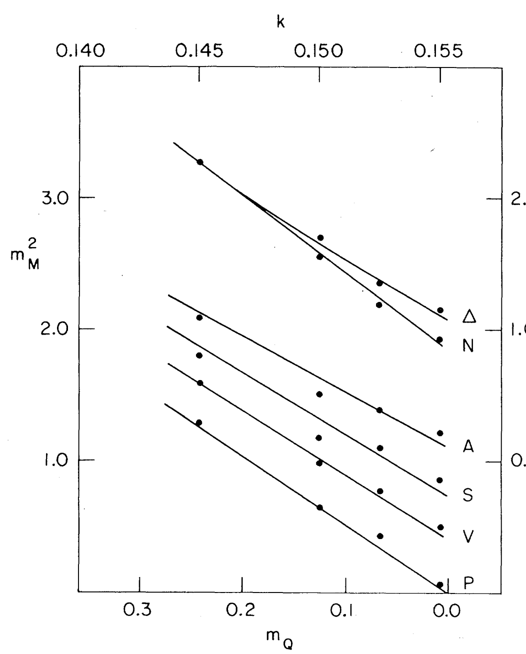

FIG. 2. Meson masses squared and baryon masses as a function of k and the bare quark mass $m_Q = (k_c - k)/2k_c^2$ obtained with use of the Wilson fermion action ($r = 1$) at $\beta = 6$. P, V, S, and A stand for pseudoscalar ($J^{PC} = 0^{-+}$), vector (1^{--}), scalar (0^{++}), and axial vector (1^{++}) masses. N and Δ stand for nucleon ($\frac{1}{2}^+$) and delta ($\frac{3}{2}^+$) masses.

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Computer Estimates of Meson Masses in SU(2) Lattice Gauge Theory

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It is shown that in an SU(2) lattice gauge theory, in the approximation where internal quark closed loops are neglected, chiral symmetry is broken. With use of partially conserved axial-vector current f_π, the bare masses of the u and d quarks, and the ρ and δ masses are estimated.

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Recently some progress has been made in numerical simulations of theories with fermions.[1-3] Although in a complete computation the effects of fermionic closed loops must be taken into account, a reasonable estimate of the hadron spectrum can be obtained by eliminating all internal quark loops (quenched case, see Ref. 2). In this way the Zweig rule is enforced for all flavors. In this note we present a study of chiral-symmetry breaking and of the π, ρ, and δ masses for the SU(2) gauge theory in the quenched approximation. A similar study for the SU(3) gauge theory, including also baryons, can be found in Ref. 4. The results obtained are rather satisfactory.

Let us begin discussing our strategy in the continuum case; later we will adapt it to the lattice version of the model. We consider the fermionic Euclidean action

$$S_f = \int d^D x\, \bar\psi (\not{D} + m)\psi\,, \tag{1}$$

where D_μ is the covariant derivative in presence of a gauge field A_μ. If $G(x,0|A)$ is the fermionic Green function with A_μ as background, and $d\mu[A]$ is the probability distribution of the field A (normalized to 1), the following relations hold:

$$\langle \bar\psi(0)\psi(0)\rangle = \int d\mu[A]\,\mathrm{Tr}[G(0,0|A)],$$
$$\langle \bar\psi(x)\gamma_5\psi(x)\bar\psi(0)\gamma_5\psi(0)\rangle = \int d\mu[A]\,\mathrm{Tr}[G(x,0|A)G^*(x,0|A)]. \tag{2}$$

