

Composite Leptons at the LHC

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ABSTRACT

In some models of electro-weak interactions the W and Z bosons are considered composites, made up of spin-one-half subconstituents. In these models a spin zero counterpart of the W and Z boson naturally appears, whose higher mass can be attributed to a particular type of hyperfine spin interaction among the various subconstituents. Recently it has been argued that the scalar state could be identified with the newly discovered Higgs (H) candidate. Here we use the known spin splitting between the W/Z and H states to infer, within the framework of a purely phenomenological model, the relative strength of the spin-spin interactions. The results are then applied to the lepton sector, and used to crudely estimate the relevant spin splitting between the two lowest states. Our calculations in many ways parallels what is done in the $SU(6)$ quark model, where most of the spin splittings between the lowest lying baryon and meson states are reasonably well accounted for by a simple color hyperfine interaction, with constituent (color-dressed) quark masses.

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1 Introduction

The idea that weak vector bosons and perhaps even leptons and quarks are composite is not new [1-11]. Recently such composite models have been given new impetus by the experimental discovery of a new Higgs-like particle [12, 13] with a mass significantly above the known weak vector bosons. In [1] it was suggested that the new spin-zero state could be identified as the spin partner of the W and Z bosons, whose higher mass could arise because of the non-trivial nonperturbative dynamics of the postulated subconstituents.

The question then arises of how such a scenario could be tested in practice, and what notable observable differences would arise when compared to the Standard Model. One obvious consequence of the existence of subconstituents is the deviation from point-like behavior at sufficiently high energies and the appearance of form factors, just as in the case of the ρ meson and most other hadrons composed of quarks. Nevertheless the experimental measurement of form factors is notoriously time-consuming and requires both high energies (to probe in detail the inner structure) and high statistics.

Another option is to pursue the analogy with the quark model, and derive a number of predictions that can be obtained in a rather straightforward way from the nature of the constituents and their color, flavor and spin wave functions. While these approaches have enjoyed some degree of success for QCD (summarized below), one additional obstacle in the case of composite weak vector bosons is represented by the fact that virtually nothing is known about the nonperturbative ground state of confining chiral gauge theories. The main reasons being that it is not easy to put chiral fermions on the lattice, and also the rather serious issue that the fermion determinant is generally complex due to the anomaly. In one of the simplest contexts the problem arises because the fermion determinant

$$\det i\mathcal{D}_L = \left[\det(-\mathcal{D}_L^2 + \delta m^2) \right]^{1/2} \quad (1)$$

is not real, since the operator

$$-\mathcal{D}_L^2 = -D_L^2 + \frac{1}{2} g \sigma_{\mu\nu} F^{\mu\nu} \quad (2)$$

is not Hermitean. Here $\sigma_{\mu\nu} = \frac{1}{2i}[\sigma_\mu, \sigma_\nu]$ with $\sigma_\mu = (1, \boldsymbol{\tau})$ and $-D_L^2 = -\mathbf{D}^2 + i D_0 \boldsymbol{\tau} \cdot \mathbf{D}$; the second order formulation is used to avoid the notorious lattice fermion doubling problem. Since chiral symmetry is no longer explicit, the δm^2 counterterm needs to be fine-tuned so as to obtain a purely left-handed fermion. Also, while gauge invariance can be maintained in the second order lattice formulation, the same might not be true for Lorentz invariance, which should hopefully nevertheless

be recovered in the continuum limit. For an early perspective on these and other subtle issues associated with the nonperturbative formulation of chiral gauge theories see, for example, [14].

2 Hyperfine Interaction

We now turn to the formulation of a simple model used later to account for the spin splittings of the observed bound states. In *QED* for two charged particles with magnetic moments $\mu_i = (e_i/2m_i)\sigma_i$ and charges $e_i e_j \equiv 4\pi\alpha$ one has

$$\Delta E = \frac{8\pi}{3} \frac{\alpha}{m_i m_j} |\psi(0)|^2 \mathbf{S}_i \cdot \mathbf{S}_j . \quad (3)$$

This applies to *s*-wave states only, for which $|\psi(0)|^2 \neq 0$. For the ground state of Hydrogen one has for the wave function at the origin

$$|\psi_{100}(0)|^2 = \frac{1}{\pi a_0^3} \quad (4)$$

for a Bohr radius $a_0 \equiv \hbar^2/m e^2$. Thus for the non-relativistic Hydrogenic case $|\psi_{100}(0)|^2 \simeq m^3$ where m is the reduced mass. For the relativistic Dirac equation case the relevant results were obtained in [15].

Similar formulas can be written down for the case of *QCD*. In the case of the color magnetic interaction of quarks one has instead, from single gluon exchange,

$$\Delta E(q\bar{q}) = \frac{32\pi}{9} \frac{\alpha_S}{m_i m_j} |\psi(0)|^2 \mathbf{S}_i \cdot \mathbf{S}_j , \quad (5)$$

for $q\bar{q}$ pairs in a color singlet meson, and

$$\Delta E(qq) = \frac{16\pi}{9} \frac{\alpha_S}{m_i m_j} |\psi(0)|^2 \mathbf{S}_i \cdot \mathbf{S}_j , \quad (6)$$

for qq pairs in a color singlet baryon. In either case the coupling is proportional to α_S , with the different numerical coefficients due to color factors as they apply to the relevant color group representation [16]. For the above formula to be useful, the parameters m_i are taken to be the dressed (or constituent) quark masses, the latter reflecting the dressing of the current algebra bare quark states by a large virtual gluon cloud. The bound state wave function at the origin can be estimated crudely in a non-relativistic potential model, but in the end it would require a complex nonperturbative (and relativistic) calculation, and for practical purposes it is taken instead as a free parameter.

For two spins with total spin $\mathbf{S} = \mathbf{s}_i + \mathbf{s}_j$ one has in general

$$\mathbf{s}_i \cdot \mathbf{s}_j = \frac{1}{2} [S(S+1) - s_i(s_i+1) - s_j(s_j+1)] = \begin{cases} -\frac{3}{4} & \text{for } S = 0 \\ +\frac{1}{4} & \text{for } S = 1 \end{cases} . \quad (7)$$

This last result can then be used to estimate the spin splittings for mesons [17, 18]. The wave function at the origin is then related to hadronic quantities such as f_ρ , the ρ meson decay constant. One important aspects of vector-like $SU(N)$ gauge theories is that chiral symmetry is dynamically broken. Instead of the free field (or perturbative) result

$$\langle \bar{\psi}\psi \rangle \simeq m_q^3 \quad (8)$$

with m_q the (current algebra) light quark mass, one instead has the nonperturbative QCD result

$$\langle \bar{\psi}\psi \rangle \simeq \Lambda_{\overline{MS}}^3 \quad (9)$$

which shows the non-vanishing nature of the QCD fermion condensate in the chiral limit $m \rightarrow 0$ [19]. In more practical terms, it is known that the condensate has the value $\langle \bar{\psi}\psi \rangle \approx (300 \text{ MeV})^3$, either inferred from a direct lattice calculation, or from the standard $PCAC$ relations involving the pion and current algebra quark masses.

In the case of three spins, where now $\mathbf{S} = \mathbf{s}_i + \mathbf{s}_j + \mathbf{s}_k$, one has instead

$$\sum \mathbf{s}_i \cdot \mathbf{s}_j = \frac{1}{2} [S(S+1) - 3s(s+1)] = \begin{cases} -\frac{3}{4} & \text{for } S = \frac{1}{2} \\ +\frac{3}{4} & \text{for } S = \frac{3}{2} \end{cases} . \quad (10)$$

The latter is of course useful for the case of baryons, where it can be used both for the baryon octet ($s = \frac{1}{2}$) and decuplet ($s = \frac{3}{2}$).

In practice it is known that both sets of formulas give a reasonably good description of the (non-singlet) $S = 0$ (pseudoscalar) and $S = 1$ (vector) meson multiplets, and an equally reasonable description of the baryon octet ($s = 1/2$) and decuplet ($s = 3/2$), with some slight modifications to account for the fact that for some baryons, like the Λ , the large quark mass difference (u/d vs. s) plays a significant role [17]. In fact, the above formulae work much better than expected. They reproduce the known light meson spectrum to within a few percent, and the light baryon spectrum (octet and decuplet) to a percentage or better. The only exception of course are the isoscalar mesons (η, η') which are affected by a large mixing with light pseudoscalar glueballs (the so-called $U(1)$ QCD axial anomaly problem), and therefore require additional input in the form of suitable flavor mixing matrices.

3 Composite W^\pm and Z^0 Bosons

In a number of composite vector boson scenarios the W and Z weak vector bosons are made up of spin- $\frac{1}{2}$ subconstituents [1]. In these theories it is generally assumed that the leptons, and perhaps even the quarks, are composite. Here we will consider, for simplicity, what we regard as one of the simplest scenarios, where the leptons are composed of three spin- $\frac{1}{2}$ elementary constituents, possibly not even exactly of the same mass (hhh , and $\bar{h}\bar{h}\bar{h}$ for their antiparticles). Furthermore, we will assume that the W and Z bosons are made of six of the same elementary constituents, three of them particles and the rest antiparticles ($hhh\bar{h}\bar{h}\bar{h}$). Later on we will also look, for completeness, at the case of W 's and Z 's made out of ($hh\bar{h}\bar{h}$) or even ($h\bar{h}$).

For concreteness, and in a first approximation, we will assume that all subconstituents have roughly the same mass, and that the binding in hypercolor singlet states occurs due to a hypercolor force based on the group $SU(N)$. Not much is known about the nonperturbative ground state of chiral gauge theories, and therefore about the nature of the effective hypercolor spin interactions, or about the hypercolor dressing of bare (current algebra) subconstituents. In the following we will pursue the simplest assumptions in order to obtain a series of admittedly rather crude estimates for the effective spin dependent forces. A more sophisticated estimate for the bound state energies is not possible yet, given our almost complete ignorance regarding the properties of the hypercolor symmetry group, the nature of the subconstituents, and the general nonperturbative properties of chiral gauge theories.

In the following the spin interaction will be modeled after the QCD one described earlier. For N spin- $\frac{1}{2}$ objects, where now $\mathbf{S} = \sum_i \mathbf{s}_i$, one obtains

$$\sum_{i>j} \mathbf{s}_i \cdot \mathbf{s}_j = \frac{1}{2} [S(S+1) - N \cdot \frac{3}{4}] \quad (11)$$

Thus each ‘‘bond’’ contributes the same amount

$$S_{12} \equiv \langle \mathbf{s}_i \cdot \mathbf{s}_j \rangle = \frac{1}{N(N-1)} [S(S+1) - N \cdot \frac{3}{4}] \quad (12)$$

Thus for six spin- $\frac{1}{2}$ objects one has

$$\langle \sum \mathbf{s}_i \cdot \mathbf{s}_j \rangle = \frac{1}{30} [S(S+1) - 6 \cdot \frac{3}{4}] = \begin{cases} -\frac{3}{20} & \text{for } S = 0 \\ -\frac{1}{12} & \text{for } S = 1 \end{cases} . \quad (13)$$

A first-principle estimate of the spin interactions would seem to require a nonperturbative calculation in a confining chiral gauge theory. But, as alluded to previously, the nature of the effective spin interaction between subconstituents in a nonperturbatively treated chiral gauge theory is not

well understood. Here the assumption will be made that it can be written in the form of a spin-spin interaction of the type used for hadron models, with two unknown parameters characterizing the strength of the fermion-anti-fermion (α) and fermion-fermion (β) direct spin interactions.

In the following we will only consider the $l = 0$ s -wave states. Then, if indeed the $S = 0$ and $S = 1$ bosons are made out of six subconstituents ($hh\bar{h}\bar{h}\bar{h}$, three fermions and three anti-fermions), one can write

$$M(m, S) = 6m + \frac{1}{m^2} (9\alpha S_{12} + 6\beta S_{12}) \quad (14)$$

with numerical coefficients reflecting the number of fermion-fermion (6) and fermion-anti-fermion (9) interaction bonds. Here the parameter m stands for the dressed mass of the subconstituents, arising mainly from the strong hypercolor confining gauge interactions. The spin interaction is modeled, again, after QED and single-gluon exchange QCD , and α and β are taken so far as entirely free real parameters. In QED one has of course $\alpha/\beta = -2$.

For the spin zero weak boson (H) one obtains

$$M(m, 0) = 6m - \frac{27\alpha}{20m^2} - \frac{9\beta}{10m^2} \quad (15)$$

and for the spin one weak boson (W)

$$M(m, 0) = 6m - \frac{3\alpha}{4m^2} - \frac{\beta}{2m^2} . \quad (16)$$

The mass splitting is therefore given by

$$M(S = 0) - M(S = 1) = \frac{1}{5m^2} (-3\alpha - 2\beta) . \quad (17)$$

The known mass of the $S = 1$ W^\pm boson ($m_W = 80.385 GeV$) and of the $S = 0$ Higgs candidate ($m_H = 125.7 GeV$) then fixes two of the free parameters, say α and the constituent fermion mass

$$m = 3.96 GeV . \quad (18)$$

Note that we used the W boson mass instead of the Z mass (or some combination of the two), since we took into consideration the fact that the Z mass is shifted upward by the mixing with the photon, which does not however affect the W mass.

An alternate possibility, advocated explicitly in [1], is to consider the W and Z bosons as made exclusively of two spin- $\frac{1}{2}$ sub-constituents, without any further ingredients ($h\bar{h}$). If that is indeed the case, then one has simply for the fermion-antifermion pair

$$\mathbf{s}_i \cdot \mathbf{s}_j = \begin{cases} -\frac{3}{4} & \text{for } S = 0 \\ +\frac{1}{4} & \text{for } S = 1 \end{cases} , \quad (19)$$

and therefore

$$M(m, S) = 2m + \frac{\alpha}{m^2} S_{12} . \quad (20)$$

From the known W and Z masses one then obtains, not unexpectedly, a much larger value for the sub-constituent effective mass $m \approx 45.9 \text{ GeV}$, as well as a spin coupling of magnitude $\alpha/m^3 = -0.989$.

4 Composite Leptons

To fix the value of the remaining spin interaction parameter β one turns next to the leptons. For concreteness we focus here on the electron, assumed to be made out of three spin- $\frac{1}{2}$ subconstituents. As before, we will consider only the $l = 0$ s -wave states. Then, in analogy to the quark model baryon octet and decuplet, the corresponding lepton state wave function would be constructed in accordance with the hypercolor analog of the quark model non-relativistic $SU(6)$ flavor-spin symmetry. Nevertheless, here we will not need to make use of any specific details of the subconstituent wave functions, and rely instead only on the spin content of the lepton subconstituents.

Then for the lepton case one has simply

$$\mu(m, S) = 3m + \frac{1}{m^2} \cdot 3\beta S_{12} \quad (21)$$

with here

$$S_{12} = \frac{1}{6} [S(S+1) - 3 \cdot \frac{3}{4}] = \begin{cases} -\frac{1}{4} & \text{for } S = \frac{1}{2} \\ +\frac{1}{4} & \text{for } S = \frac{3}{2} \end{cases} . \quad (22)$$

After using the known value for the electron mass $m_e = 0.511 \text{ MeV}$, one is finally able to fix the parameter β as well. One finds

$$\frac{\alpha}{m^3} = -21.75 \quad (23)$$

$$\frac{\beta}{m^3} = 4.00 \quad (24)$$

and thus for the ratio

$$\frac{\alpha}{\beta} = -5.44 . \quad (25)$$

For the spin- $\frac{3}{2}$ heavy lepton one obtains the estimate

$$\mu_{3/2} = 23.74 \text{ GeV} . \quad (26)$$

Note that if one had given a zero mass to the lightest $s = \frac{1}{2}$ lepton (like a neutrino) then the above numbers would have hardly changed, and one would have still obtained $\mu_{3/2} \approx 23.74 \text{ GeV}$. The

main reason for this lies in the fact that the electron is, for practical purposes, already very *light*. One potential problem with this rather low excited lepton mass value is that it might already be largely excluded by the *LEP* data on Z^0 decays [20].

Again we can revisit here the alternate possibility of [1] where the the W and Z bosons as made exclusively of two spin- $\frac{1}{2}$ sub-constituents ($h\bar{h}$). Then the leptons and quarks contain a single fermion and also an additional hypercolor-carrying scalar (spin-zero) constituent, of unknown mass. Generally elementary scalars suffer from quadratic mass divergences, so it is unclear how such a state could generate the relatively light masses of quarks and leptons. To make further progress, we will assume here that such a scalar state is itself a bound state, or condensate, of either two fermions (hh) or a fermion-antifermion pair ($h\bar{h}$). Then, if the spin interactions between the three sub-constituents is treated on an equal basis, one has for the first case $\beta/m^3 \approx 4$ and in the second case $\beta/m^3 \approx 14$, where m and α are fixed, as before, by the W mass. In either case though one finds the same result for the spin- $\frac{3}{2}$ heavy lepton, namely $\mu_{3/2} \approx 275.1 GeV$. This last case would suggest that the more sub-constituents are arranged into the W and Z bosons, the lighter the heavy spin- $\frac{3}{2}$ lepton can be made. So, one way of viewing our (admittedly very simple) results is that if no light $s=\frac{3}{2}$ lepton (we find about $24 GeV$) is observed, that would exclude the case of a W or Z made out of many (six) sub-constituents, and favor instead a scenario where fewer (say two) sub-constituents make up the weak vector bosons. For completeness we will also quote here the excited lepton mass obtained if one assumes that the W and Z bosons are made of four spin- $\frac{1}{2}$ sub-constituents ($hh\bar{h}\bar{h}$). These bound states are similar to the four-quark exotic states of *QCD* considered in [21]. Then one has $\mu_{3/2} \approx 86.59 GeV$, which lies in between the two previous cases. Again it would seem that this value could already be excluded by the *LEP* data on Z^0 decays.

Let us note here that in the quark model the main source of uncertainty in predicting the splittings in the baryon multiplets from the meson octet, or vice versa, is the fact that the wave function at the origin is not quite the same for the two type of states; in fact it is known that then r.m.s. charge radius of mesons ($r_0 \approx 0.6 fm$) is smaller than the corresponding quantity for baryons ($r_0 \approx 0.8 fm$), which when cubed gives rise to roughly a factor of two correction, and in the right direction to account for the observed hadron spin splittings. If that is the case here as well (in the sense that the r.m.s. charge radius for the electron could be significantly smaller than the corresponding quantity for the vector and scalar weak bosons, when taking into account the fact that the scalar and vector bosons contain twice as many subconstituents compared to the leptons), then this would suggest that the parameter β varies a bit depending on which states are considered (composite leptons vs. composite vector and scalar bosons). Nevertheless, the subconstituent mass

m is expected to stay the same, and β is after all determined here only from the known electron mass, so the spin splitting in the lepton sector remains unchanged, at least in the context of this rather simple model.

From a non-relativistic perspective (which might very well be totally inadequate in the present context) one has $\alpha, \beta \simeq |\psi(0)|^2$, giving therefore in light of the previous results $|\psi(0)|^2 \simeq m^3$. For quark-antiquark pairs the latter wave function value is related, via the Van Royen-Weisskopf formula and its *QCD* extensions [22], to the leptonic decay width of a vector boson,

$$\Gamma(V \rightarrow l\bar{l}) \approx \frac{16\pi \alpha^2 e_Q^2}{M_V^2} |\psi(0)|^2 [1 + \mathcal{O}(\alpha_S)] \quad (27)$$

where α here is the usual *QED* fine structure constant, α_S the strong *QCD* coupling, and e_Q the relevant quark charge in units of the proton charge. All one can say in the present context is perhaps that in the above formula, translated to the weak vector boson context, one would expect as a rough order of magnitude estimate (and not much more) $|\psi(0)|^2 \simeq m^3$, where m is the subconstituent effective mass given previously.

As far as production processes are concerned [23, 24], if this light spin- $\frac{3}{2}$ lepton indeed exists, then it should be observed along the standard decay modes for the Z , such as $Z \rightarrow e^+e^-$, with the new spin- $\frac{3}{2}$ lepton replacing the standard electron. The present models gives few prediction about the width of the new composite state, but it could be quite broad (as in the case of the ρ meson for which $\Gamma_\rho/m_\rho \approx 776 \text{ MeV}/149 \text{ MeV} \approx 5\%$).

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