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Non-Trivial Fixed Point for Quantum Gravity in Four Dimensions *

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ABSTRACT

The path integral and the phase structure of simplicial quantum gravity in four dimensions is investigated by computer simulation. With a lattice analogue of the DeWitt functional measure and a higher derivative term as a regulator, it is shown that two phases exist. In the phase of the weak field expansion (small positive G) the simplices are collapsed and no meaningful ground state exists within the context of the model. In the other phase (large G) the curvature is small and negative, and approaches zero at the critical point in a continuous fashion. From this phase, fluctuations in the curvature diverge at the critical point, while fluctuations in the volume remain bounded. The critical exponents at the transition are estimated. The results suggest the existence of a non-trivial fixed point for quantum gravity at some finite G_c.

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1. INTRODUCTION

Four-dimensional quantum gravity is a difficult theory to solve due to the unboundedness of the pure gravitational action, its non-renormalizability and non-polynomial nature[1]. Recently substantial progress has been achieved in two dimensions both in the continuum and on the lattice, where some exact results have been obtained. Understanding properties of two-dimensional lattice quantum gravity is likely to be an essential step on the way to formulating and solving a lattice theory of four-dimensional quantum gravity. In four dimensions lattice calculations are more difficult since there are more terms in the pure gravity action, there are no exact results in the full theory to compare with, and the lattice structure and the interactions are more complex. For these reasons the lattices that have been studied up to now are quite small. Furthermore there is a conceptual issue of what physical quantities should be measured, and for what boundary conditions. Only a small set of these questions have been addressed up to now, mostly pertaining to an investigation of the phase diagram and the location of possible phase transition points.

We will concentrate here on the simplicial formulation of quantum gravity also known as Regge calculus (for a review, see refs. [2,3]). One of the advantages of the approach lies in the fact that it can be formulated in any space-time dimension (including the physically relevant case of four dimensions), and that it can be shown to be classically equivalent to general relativity. Furthermore the correspondence between lattice and continuum quantities is clear, and the interpretation of the terms in the action as well as the identification and separation of, for example, the measure contribution is unambiguous[4−10]. For a more complete list of early references, see refs. [2,3]. Important issues that need to be addressed in lattice quantum gravity include the following:

Restoration of general coordinate invariance
Universality in lattice quantum gravity
Possible phases of the theory

A detailed description of the construction of the action for simplicial lattice gravity without and with matter fields can be found in the literature[4,8,9,11], and will not repeated here. Here we will recall the geometric correspondence between continuum and lattice quantities in four dimensions

$$\begin{aligned} \int d^4x\,\sqrt{g(x)} \;&\rightarrow\; \sum_h V_h \\ \int d^4x\,\sqrt{g(x)}\,R(x) \;&\rightarrow\; 2\sum_h \delta_h A_h \\ \int d^4x\,\sqrt{g(x)}\,R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}(x) \;&\rightarrow\; 4\sum_h \delta_h^2 A_h^2/V_h \end{aligned} \qquad (1.1)$$

where δ_h is the deficit angle at the hinge (triangle) h, A_h is the area of that hinge, and V_h is the volume associated with the hinge h (which is not necessarily unique, since the lattice can be subdivided geometrically in more than one way; in the following we will use the baricentric subdivision). Given reasonable geometric and positivity properties, universality is expected to lead to the same results in some continuum limit. One also notes that in the simplicial formulation, as in the continuum, the local curvature $R(x) \sim 2\delta_h A_h / V_h$ is a continuous function of the relevant edge lengths. The straightforward geometric correspondence of lattice and continuum quantities is one clear asset of the Regge calculus approach.

While simplicial quantum gravity can be formulated on a random lattice[7], it appears advantageous at least initially to adopt a regular lattice, which is much easier to work with, at least from a computational point of view. Most of the simulations of lattice gravity in four dimensions have been done for such regular lattices[2,10-13]. In most of the following we will discuss results for a simplicial complex topologically equivalent to the four-torus (periodic boundary conditions in all directions).

2. THE GRAVITATIONAL MEASURE

An important issue that needs to be addressed in lattice gravity is the problem of the gravitational measure. In the continuum the form of the measure for the $g_{\mu\nu}$ fields appears not to be unique [16-18]. The reason for the ambiguity appears to be a lack of a clear definition of what is meant by $\prod_x$ in the functional measure. It would seem that such an ambiguity persist in *all* known lattice formulation of quantum gravity. However the difference among the measures seems to be in the power of $\sqrt{g}$ in the prefactor, which corresponds to some product of volume factors on the lattice.

DeWitt has argued that the gravitational measure should be constructed by first introducing a super-metric over metric deformations, which in its simplest local form leads to the reparametrization invariant norm[16]

$$\|\delta g\|^2 = \frac{1}{2} \int d^d x \sqrt{g} \times \left[g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha} + \lambda g^{\mu\nu} g^{\alpha\beta} \right] \delta g_{\mu\nu} \delta g_{\alpha\beta} \quad (\lambda \neq -\frac{2}{d}) \tag{2.1}$$

It then gives a functional measure for pure gravity in d dimensions of the form

$$d\mu[g] = \prod_x g^{(d-4)(d+1)/8} \prod_{\mu \geq \nu} dg_{\mu\nu} \tag{2.2}$$

Another popular (pure) gravitational measure in the continuum is the Misner scale-invariant measure[17]

$$d\mu[g] = \prod_x g^{-(d+1)/2} \prod_{\mu \geq \nu} dg_{\mu\nu} \tag{2.3}$$

It is unique if the product over x is interpreted as one over 'physical' points, and coordinate invariance is imposed at one and the same 'physical' point. Other forms of the measure for the gravitational field have also been suggested, inspired by the canonical quantization approach to gravity[18]. If matter fields are present, then the gravitational measure has to be further modified[16] (in the lattice context see the discussion in [12]).

On the simplicial lattice the edge lengths should be considered as the elementary degrees of freedom, which uniquely specify the geometry for a given incidence matrix, and over which one should perform the functional integral[2,8,10]. In our earlier work we have employed the pure gravity measures

$$\int d\mu_\epsilon[l] = \prod_{\text{edges ij}} \int_0^\infty dl_{ij}^2 l_{ij}^q \; F_\epsilon[l] \tag{2.4}$$

with $q = 0, -2$, and $F_\epsilon[l]$ a function of the edge lengths with the property that it is equal to one whenever the triangle inequalities are satisfied, and zero otherwise. The parameter ϵ is introduced as an ultraviolet cutoff at small edge lengths: the function $F_\epsilon[l]$ is then chosen to be zero if any of the edges are equal or less than ϵ. The introduction of such a cutoff seems to be necessary in four dimensions[2,10], but not in two[12]. The above measure is of course correct in the weak field limit, where all continuum measures agree as well.

Since we have argued that it is not entirely clear what the measure in the continuum should be, it would seem of interest to explore the sensitivity of the results to the type of gravitational measure employed. Another class of pure gravity measures which can be written down on the lattice is obtained by considering the 'volume associated with an edge' V_{ij}, and writing for example

$$\int d\mu_\epsilon[l] = \prod_{\text{edges ij}} \int_0^\infty \; V_{ij}^{2\sigma} \; dl_{ij}^2 \; F_\epsilon[l] \tag{2.5}$$

with $2\sigma = -1/d$ for the lattice analogue of the Misner measure, and $2\sigma = (d-4)/2d$ for a lattice analogue of the DeWitt measure (note that the 'Misner' and dl/l measure share the property of being scale invariant). One would like to see how the results depend on the form of the measure and on σ. Our results in two dimensions suggest that different measures, within a certain universality class, will give the same results for infrared sensitive quantities, like correlation functions at large distances and critical exponents. We believe though that the lattice path integral might not be meaningful for certain values of σ. We have found in particular in two dimensions that if σ is too negative, then the measure factors tend to favor configurations of triangles which are long and thin, with a small area and a large perimeter. In this case the lattice tends to degenerate into a lower-dimensional manifold, a situation far from the desired continuum limit, and which one would like to avoid. In two dimensions we have systematically studied the sensitivity of the results for infrared sensitive quantities, like the fractal dimensions, on the measure. In four dimensions, exploring the sensitivity of the results to different values of σ is beyond the scope of this work. Here we will present results that have been obtained for the DeWitt measure (dl^2), corresponding to $\sigma = 0$ in eq. (2.5)

3. THE GRAVITATIONAL ACTION

We have employed the discrete analog of the higher derivative action in the form

[2,8,10]

$$I = \sum_{hinges\ h} \left[\lambda\, V_h - k\, \delta_h A_h + 4b\, \frac{A_h^2 \delta_h^2}{V_h} \right] + \frac{1}{3}(a-4b) \sum_{sites\ p} V_p \sum_{hinges\ h,h' \supset p} \epsilon_{h,h'} \left[\omega_h \frac{A_h \delta_h}{V_h} - \omega_{h'} \frac{A_{h'} \delta_{h'}}{V_{h'}} \right]^2 \tag{3.1}$$

(the numerical factor $\epsilon_{h,h'}$ is equal to 1 if the two hinges h, h' have one edge in common and -2 if they do not). The motivations leading to the above action are discussed in detail in [2,8,10] and will not be repeated here. The introduction of the higher derivative terms is motivated by the fact that the resulting extended theory of gravity is renormalizable in four dimensions[19–23]. To simplify things further we have actually set $a = 4b$ below. The higher derivative term proportional to b acts as a regulator and allows us to establish contact with results for continuum higher derivative theories.

In four dimensions the classical continuum limit is taken by requiring that the local curvature be small on the scale of the local lattice spacing, which is equivalent to imposing

$$\left| \frac{A_h \delta_h}{V_h} \right| \ll \frac{1}{\sqrt{V_h}} \tag{3.2}$$

and implies

$$\frac{A_h^2 \delta_h^2}{V_h} \ll 1 \tag{3.3}$$

This condition can be met by having the coefficient of the curvature squared terms large; otherwise the results are expected to depend on the detailed structure of the ultraviolet cutoff (i.e. choice of lattice structure and lattice transcription of the continuum action). Quantum mechanically it is known that the continuum limit has to be taken at a (non-trivial) fixed point of the renormalization group, if one can be found.

In the classical continuum limit the above action is equivalent to the continuum higher derivative action

$$I = \int d^4x \sqrt{g} \left[\lambda - \frac{k}{2} R + b\, R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \frac{1}{2}(a-4b)\, C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right] \tag{3.4}$$

with a cosmological constant term (proportional to λ), the Einstein term ($k/2 = 1/16\pi G$, where G is the bare Newton constant), and two higher derivative terms with additional dimensionless coupling constants a^{-1} and b^{-1}. It is known that the above action[21] leads to a renormalizable[22] (and asymptotically free[23]) theory in four dimensions, and for sufficiently large λ it can be shown that the action is bounded from below. It is also known that higher derivative theories tend to have problems with unitarity in the weak field limit, and these problems will have to be addressed by a full non-perturbative calculation[23].

4. PHASES OF PURE GRAVITY

Non-perturbative studies of quantum gravity are motivated by the search for a non-trivial fixed point in four dimensions[20,2]. One of the advantages of the lattice approach is that it allows non-perturbative calculations. The first numerical simulations for lattice gravity, with the above action, were discussed in [2,10]. As in the two-dimensional case, the lattice was chosen to be regular, and built out of rigid hypercubes. This choice is clearly not unique, and is dictated mostly by a criterion of simplicity, with the advantage that such a lattice can be used to study rather large systems with little algorithm modification. For the same reason up to now only the action with $a = 4b$ only (no Weyl term) was considered.

Let us now discuss some of the methods and the results obtained. In our previous numerical simulations the lattice was chosen of size $L \times L \times L \times L$ with $15L^4$ edges, and only the cases $L = 2$ (240 edges), $L = 4$ (3840 edges), and exceptionally $L = 8$ (61440 edges), were considered. Here we will present rather accurate results for all three lattice sizes, in order to provide an estimate for the volume dependence of the results. Periodic boundary conditions were used, and the topology was therefore restricted to a hypertorus; other topologies can in principle be studied by changing the boundary conditions.

In the case in which all the couplings are zero ($a = b = k = \lambda = 0$) the total action is zero, and variations in the edge lengths are only constrained by the non-trivial gravitational measure of eq. (2.5). Let us introduce here therefore some definitions. Quantities of interest which have been computed include the average curvature $\mathcal{R}$

$$\mathcal{R} \; = < l^2 > \frac{< 2 \sum_h \delta_h A_h >}{< \sum_h V_h >} \tag{4.1}$$

and the average curvature squared $\mathcal{R}^2$

$$\mathcal{R}^2 \; = < l^2 >^2 \frac{< 4 \sum_h \delta_h^2 A_h^2 / V_h >}{< \sum_h V_h >} \tag{4.2}$$

which are here both dimensionless quantities, since they have been expressed in units of the average edge length. Remarkably one finds that at strong coupling the system tends to develop an average negative curvature[2,10].

When the parameters λ, k and a in eq. (3.1) take non-zero values, then one finds a sudden very sharp transition between a state in which the curvature is small and one in which it is very large, presumably a reflection of the unbounded fluctuations in the conformal modes found in the continuum. For example for the dl/l measure, $\lambda = 1$ and for small higher derivative coupling ($a = 4b = 0.005$), the average curvature $\mathcal{R}$ jumps from a small negative value to a large positive one for $k \simeq 2.0$ or greater. A similar type of transition is found if k is kept fixed and λ is varied[2,10]. The jump in $\mathcal{R}$ is so large, that it could be perhaps indicative of a discontinuous transition, and it was suggested that the transition is connected with the lowest eigenvalue of the quadratic edge length fluctuation matrix around the saddle point becoming zero and then negative, as in the case of the regular tessellation of the four-sphere α_5[9,8]. Indications of a discontinuous transition were later found also for the pure Einstein action in a fixed volume ensemble[13], as well as in the hypercubic lattice formulation of gravity[24–27].

To further explore these issues, which had been left unanswered by our previous study, we have performed a number of large scale simulations of pure gravity, using the dl^2

('DeWitt') measure, and employing lattices of size up to 8^4 (with 61440 edges) (we have also done some short runs on 16^4 lattices (with 983040 edges), but we will not discuss the results here in detail since the statistics is at this point still rather low). Let us emphasize that at this point the nature of the results is still rather preliminary. A more detailed description of the results will be given in a separate publication[28]. The lengths of our runs typically vary between 24000 Metropolis Monte Carlo iterations on the 2^4 lattice, 6000 iterations on the 4^4 lattice, and at least 1200 iterations on the 8^4 lattice. In all cases the starting lattices were duplicated copies of the smaller lattice, for each k, and as is customary additional thermalization sweeps were performed for each lattice and value of k, after duplicating the lattice. In Fig. 1 we show the distribution of curvatures $\delta_h A_h \sim \sqrt{g}\, R(x)$ on the 16^4 lattice, for $\lambda = 1$, $k = 0.2$ and $a = 0.005$.

It is of interest to explore the dependence of the results on the measure, and to investigate in more detail the transition between the 'smooth' and the 'rough' phases of spacetime described above. Besides the quantities $\mathcal{R}$ and $\mathcal{R}^2$ discussed before, we have therefore also computed the lattice analogues of the fluctuations in the local curvatures

$$\chi_{\mathcal{R}} = \frac{1}{< \sum_h V_h >} [< (2 \sum_h \delta_h A_h)^2 > - < 2 \sum_h \delta_h A_h >^2] \tag{4.3}$$

and of the fluctuations in the local volumes

$$\chi_V = \frac{1}{< \sum_h V_h >} [< (\sum_h V_h)^2 > - < \sum_h V_h >^2] \tag{4.4}$$

A divergence in the fluctuation is then indicative of long range correlations (a massless particle) in the relevant channel. The results obtained for the dimensionless average curvature $\mathcal{R}$ are shown, for different lattice sizes, in Fig. 2. The statistical errors in $\mathcal{R}$ are estimated by the usual binning procedure, and represent one standard deviation. One finds that as long as one does not move too close to k_c, the autocorrelations are contained.

One notes that as k is varied, the curvature appears to go to zero at some finite value k_c. In the particular case studied, namely for $\lambda = 1$ and $a = 0.005$, one finds $k_c = 0.239 \pm 0.012$ on the largest lattice ($L = 8$), with values compatible within errors on the smaller lattices. For k close to, but less than, k_c one can then write

$$\begin{aligned} \mathcal{R} &\underset{k \to k_c}{\sim} A_{\mathcal{R}}\, (k_c - k)^{\delta} \\ \chi_{\mathcal{R}} &\underset{k \to k_c}{\sim} A_{\chi}\, (k_c - k)^{\delta - 1} \end{aligned} \tag{4.5}$$

where δ is a universal exponent characteristic of the transition. Performing a simultaneous fit in A, k_c and the exponents, one finds $\delta = 0.61 \pm 0.04$ ($A_{\mathcal{R}} = -3.68 \pm 0.07$) from the average curvature, and $\delta - 1 = -0.35 \pm 0.22$ from the curvature susceptibility (see Fig. 3). From the curvature susceptibility we estimate $k_c = 0.24 \pm 0.06$, in agreement with the previous estimate from the average curvature, but with a much larger error due to the difficulty of estimating the susceptibility accurately on the larger lattice, since the runs are still a bit too short, especially close to k_c.

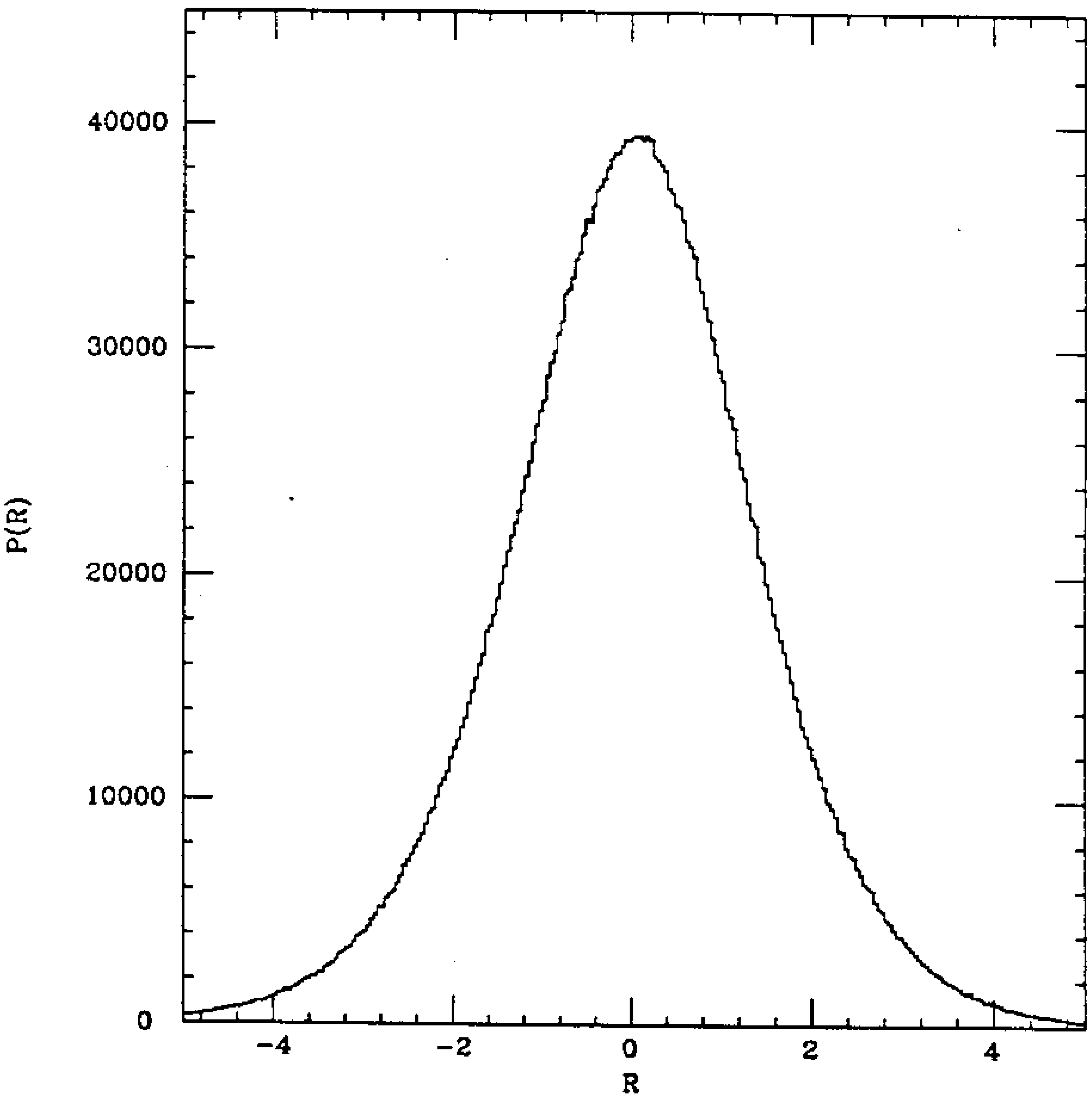

Figure 1: Histogram of the distribution of curvatures $\delta_h A_h \sim \frac{1}{2}\sqrt{g}\, R$ on a lattice with $15 \times 16^4 = 983040$ edges, and for $\lambda = 1$, $k = 0.2$ and $a = 0.005$ (dl^2 measure).

In analogy to the two-dimensional case one can define a critical exponent γ_χ, which determines the leading volume correction to the curvature

$$\frac{k}{4} \frac{< \int \sqrt{g}\, R >}{< \int \sqrt{g} >} \underset{V \to \infty}{\sim} c + \frac{\gamma_\chi - 2}{V} + \cdots \tag{4.6}$$

Near k_c one finds that γ_χ is, for the torus, quite close to and consistent with the value 2, namely $\gamma_\chi - 2 = 0.023 \pm 0.010$. This result is reminiscent of the two-dimensional case, where γ_χ is exactly two for the torus.

The above results are consistent with the picture of a vanishing curvature and a divergent curvature fluctuation, at the same value of k_c. We should point out though that

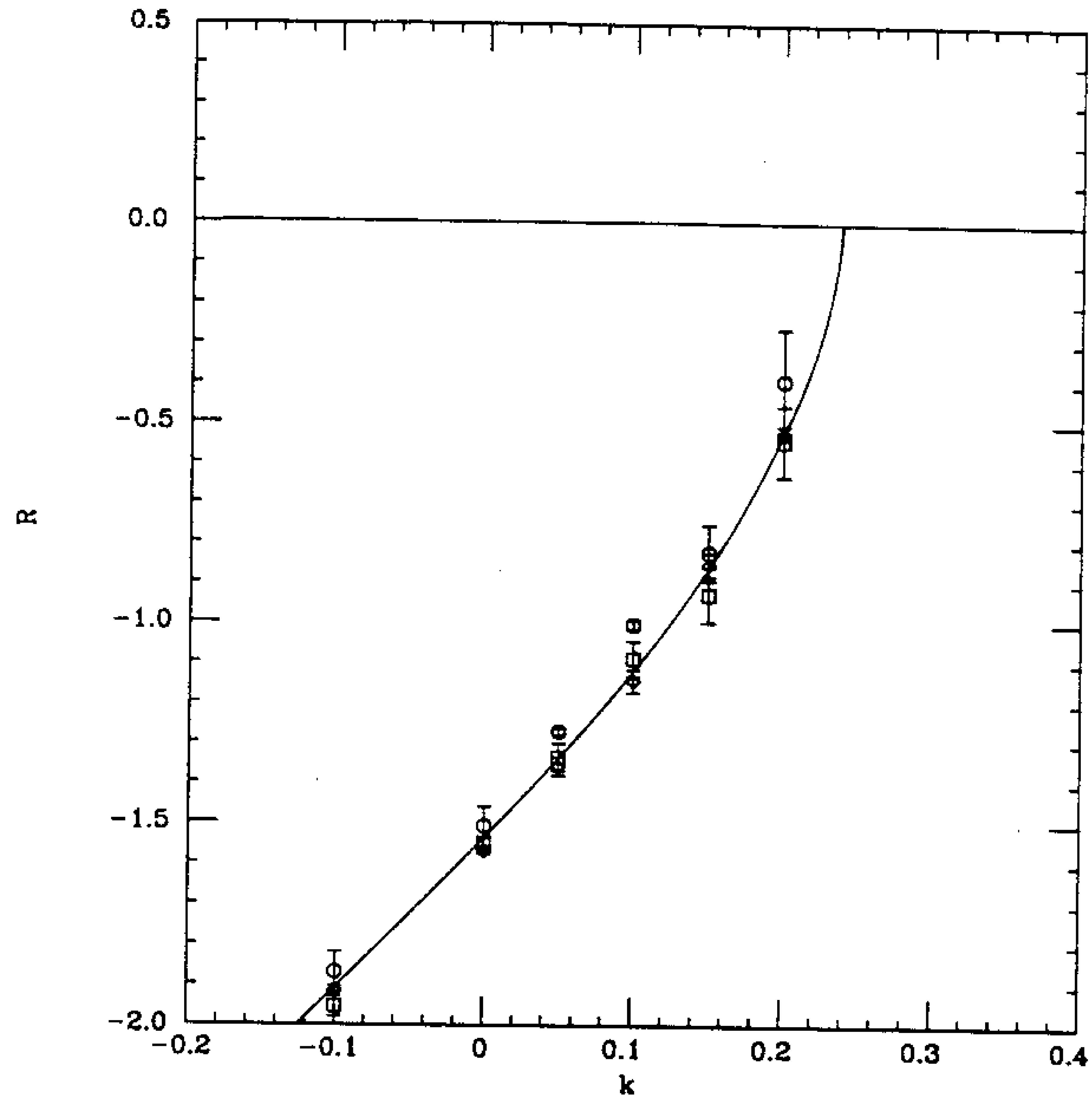

Figure 2: Average curvature $\mathcal{R}$ as a function of k, for $\lambda = 1$ and $a = 0.005$ (dl^2 measure). The circles refer to $L = 2$, the squares to $L = 4$, and the diamonds to $L = 8$.

one cannot entirely exclude a discontinuous (first-order) transition at k_c, with a rather small discontinuity. But from our results there is clearly no evidence either for such a discontinuous transition. We notice that the numerical solution gives correctly a negative sign for the average curvature, which is needed for $k < k_c$ in order to have a positive fluctuation $\chi_{\mathcal{R}}$. Also the average curvature becomes complex for $k > k_c$, reflecting the fact that the theory becomes unstable in that regime, and that the curvature is really infinite (or very large on a large lattice) in that phase. If we compute the volume susceptibility, we find on the other hand that it approaches a finite value at k_c, suggesting the absence of critical volume fluctuations (see Fig. 4). These appear to be desirable properties in a theory supposedly describing gravity, where the excitations in the continuum are expected

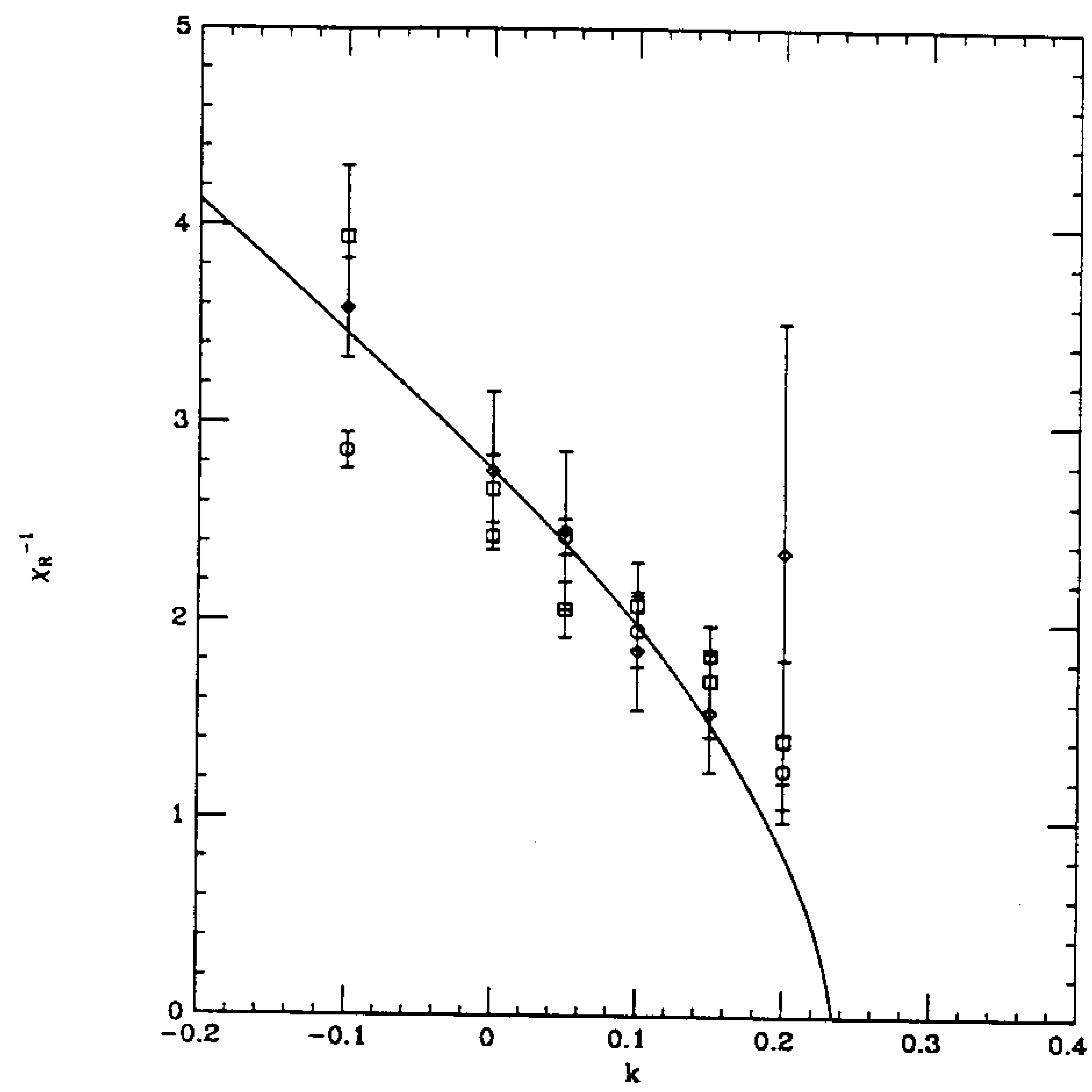

Figure 3: Curvature fluctuation $\chi_{\mathcal{R}}$ as a function of k, for the same parameters as in Fig. 2.

to be massless gravitons, without any massless scalar particles and no massless volume density fluctuations.

On the other side of the transition ($k \geq k_c$) one has that the curvatures are infinite, since the simplices collapse into degenerate configurations with very small volumes ($< V_h > / < l^2 >^2 \sim 0$). This is the region of the weak field expansion ($G \to 0$), and it is therefore not surprising that it has difficulties in extending to the region where a sensible path integral for pure gravity can be defined. A qualitative picture of the phase diagram for pure gravity with our lattice action is sketched in Fig. 5.

It would appear that close to the transition one is dealing with two rather different length scales. In an attempt to clarify the nature of this transition, one can define first a

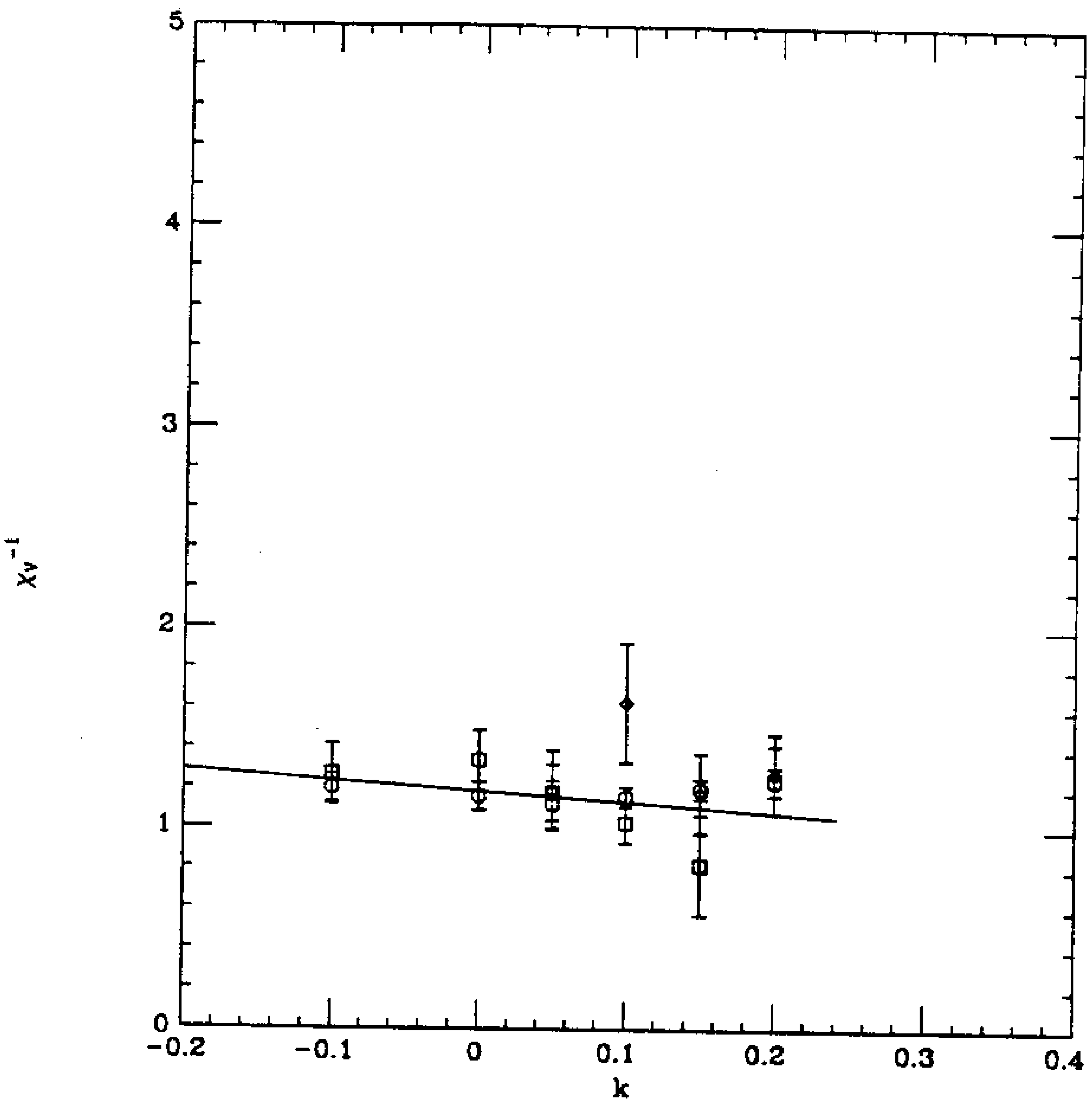

Figure 4: Volume density fluctuation χ_V as a function of k, for the same parameters as in Fig. 2.

length scale R_0 associated exclusively with the space-time average of the curvature, and therefore related to some average curvature radius,

$$\frac{< 2\sum_h \delta_h A_h >}{< \sum_h V_h >} \equiv \frac{1}{R_0^2} \sim \left(\frac{4\lambda}{k}\right)_{eff} \tag{4.7}$$

As one approaches the fixed point at k_c, this length scale becomes very large. Naturally another length scale M_0^{-1} can then be associated with the average volume (per hinge or per site)

$$\frac{N}{\sum_h V_h} \equiv \frac{M_0}{R_0^3} \sim \lambda_{eff} \tag{4.8}$$

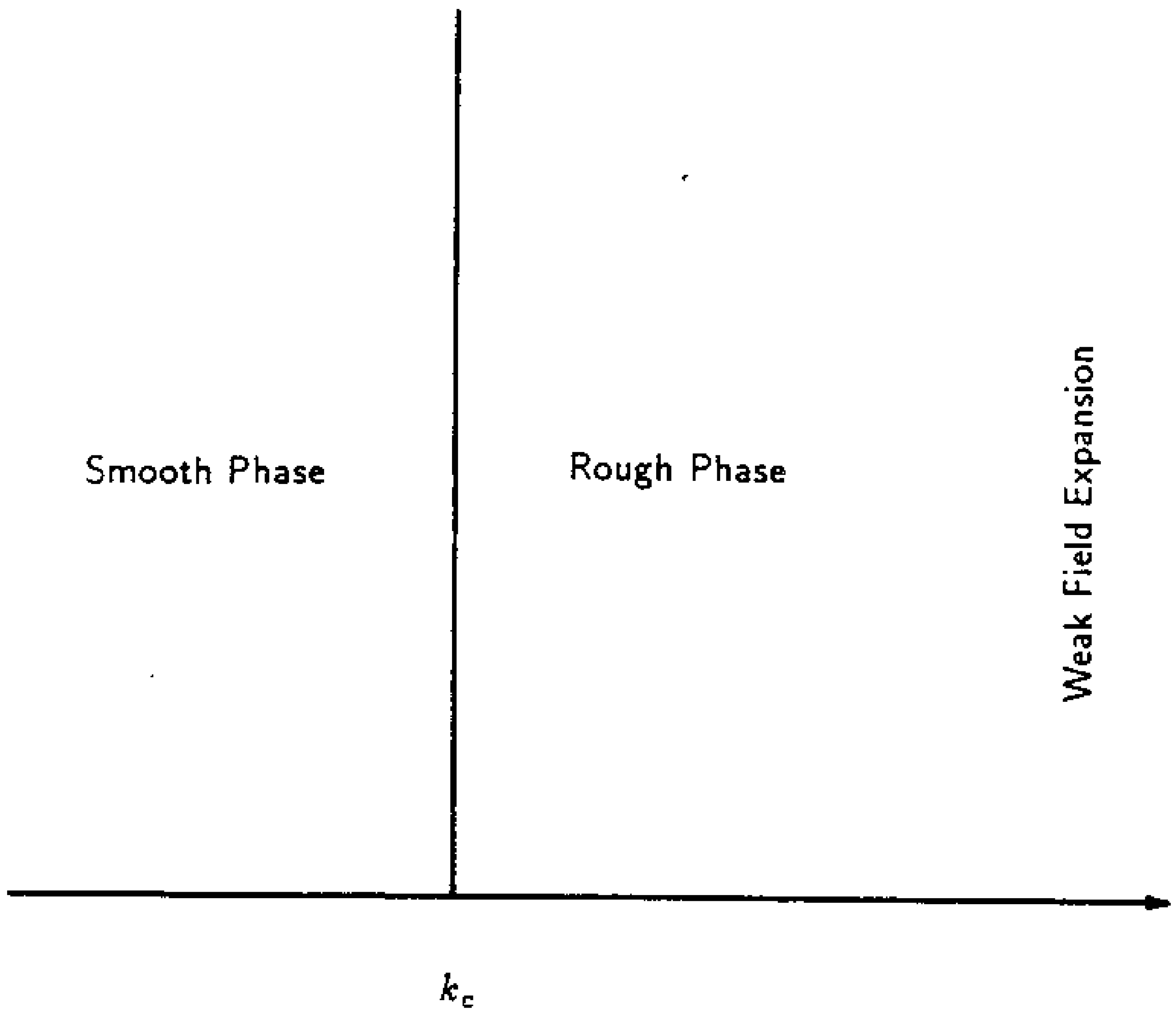

Figure 5: Phase diagram for pure 4-d gravity.

Close to and below the critical point this particular combination approaches some constant value. From the numerical solution one then obtains for R_0

$$R_0 \underset{k \to k_c}{\sim} A_{R_0} \, (k_c - k)^{-\delta/2} \tag{4.9}$$

with $A_{R_0} = 1.17 \pm 0.03$ and $\delta/2 = 0.31 \pm 0.03$, and also for M_0 one gets

$$M_0 \underset{k \to k_c}{\sim} A_{M_0} \, (k_c - k)^{-3\delta/2} \tag{4.10}$$

with $A_{M_0} = 0.216 \pm 0.005$. Here we have used the fact that for the dl^2 measure the volume per site (or per hinge), as well as the average edge length, approaches a constant different

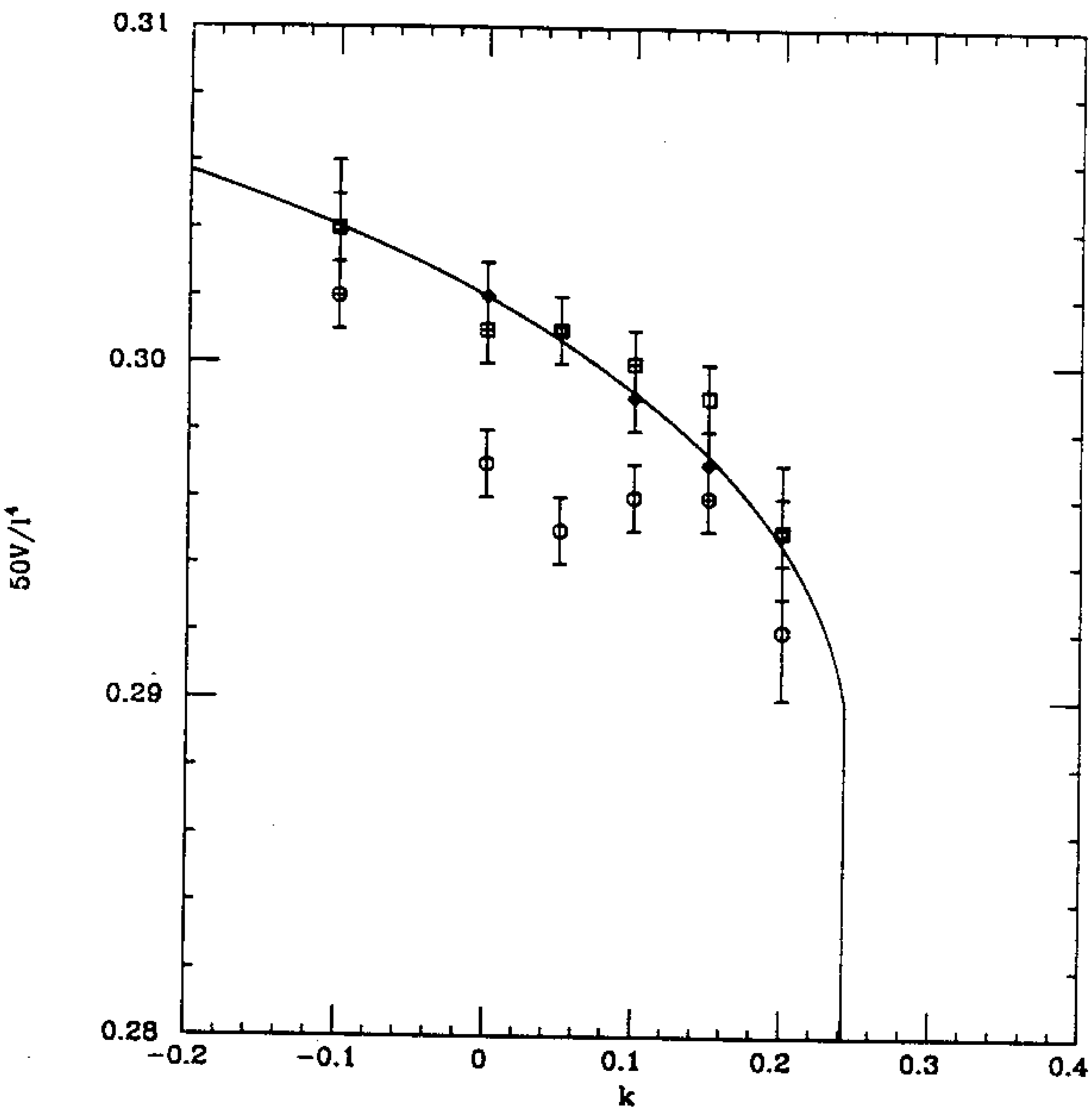

Figure 6: Average volume in unit of the average edge length $50 < V_h > / < l^2 >^2$ as a function of k, for the same parameters as in Fig. 2.

from zero as k tends toward k_c from below. So does their ratio $50 < V_h > / < l^2 >^2 \simeq (7.48 \pm 0.01)/(2.25 \pm 0.02)^4 \simeq 0.293 \pm 0.002$, shown in Fig. 6 (The factor of 50 arises since there are 50 hinges or triangles per site for our regular hypercubic lattice). This ratio is zero in the other phase, indicating a sharp discontinuity at $k = k_c$. The fact that the volume is discontinuous as k_c is consistent with the fact that the volume fluctuation approaches a finite value, and does therefore not diverge, at k_c.

Our results therefore suggest that one is dealing simultaneously with a very large (R_0) and a very small (M_0^{-1}) length scale, associated with the curvature and density of the 'universe', respectively. In particular we conclude that the *dimensionless* ratio of R_0^{-1}

over M_0 is given by

$$\frac{1}{R_0 M_0} \sim \left(\frac{4\lambda}{k^2}\right)_{eff} \underset{k \to k_c}{\sim} (k_c - k)^{2\delta} \tag{4.11}$$

As k approaches the fixed point k_c, this quantity vanishes with a power which we estimate at about $2\delta = 1.22 \pm 0.08 > 0$, in agreement with the observed fact that the renormalized cosmological constant is very small, once it is expressed in the appropriate units. In our model this is simply a consequence of the fact that the curvature is small in units of the volume, as long as one does not cross over into the 'rough' phase of gravity ($k > k_c$). There no sensible ground state seems to exist, at least within the context of our model.

The results presented above for the average curvature $\mathcal{R}$ are not inconsistent with known results within the weak field expansion in the continuum. Substituting $k^{-1} = 8\pi G$, and setting $k_c = c\Lambda^2$, where c is a constant independent of k, and Λ the ultraviolet cutoff (here of the order of the average inverse lattice spacing $\sim < l^2 >^{-1/2}$), one obtains

$$\begin{aligned} \mathcal{R} \sim & A_{\mathcal{R}} \left(\frac{-1}{8\pi G}\right)^{\delta} (1 - c\Lambda^2 8\pi G)^{\delta} \\ \sim & A_{\mathcal{R}} \left(\frac{-1}{8\pi G}\right)^{\delta} \left[1 + \delta\, c\Lambda^2(-8\pi G) + \frac{\delta(\delta-1)}{2} (c\Lambda^2)^2 (-8\pi G)^2 + \cdots\right] \end{aligned} \tag{4.12}$$

One can see that, besides $\mathcal{R}$ possibly not being analytic at $G = 0$, an expansion in powers of G involves increasingly higher powers of the ultraviolet cutoff Λ, as expected from a theory which is not perturbatively renormalizable in G. The above results are therefore not inconsistent with what is known in the continuum.

More detailed and careful computations are needed to better understand these interesting issues, and it seems that correlations should be computed in order to determine the excitation spectrum. It also of course of interest to understand better the phase diagram of the theory, and how the results depend on the dimensionless higher derivative coupling a. One would expect that the exponents do not depend on a, unless a new phase occurs for large enough a. It would also be interesting to make contact with the perturbative results for large a obtained within the weak field approximation. Similarly we would expect that the results for the exponents should not depend on small variations in the gravitational measure, as is the case in two dimensions, but these issues need still to be explored in four dimensions. Matter fields (bosonic or fermionic) can also be included, and they could play a role similar to the higher derivative terms in stabilizing the ground state. In this last case one could distinguish between external fixed matter distributions, and the effects of dynamical matter loops.

5. CONCLUSIONS

In the preceding sections we have discussed results relevant for a model of simplicial quantum gravity in four dimensions. It is characteristic of our model that the variations in the geometry of space are described by fluctuating edge lengths on a lattice with fixed coordination number.

In four dimensions we have studied in some detail a model for pure lattice gravity, with a higher derivative term acting as a regulator, and using a lattice analogue of the

DeWitt measure. In our earlier work we had found on small lattices a rather sudden transition between a 'smooth' and a 'rough' phase of pure gravity. The detailed nature of that transition had remained unclear. We have now extended our earlier results on the phase transition to a different measure, as well as to much larger lattices, and have shown that the transition appears to be continuous if it is approached from the smooth phase (small k or large G), which is characterized by a negative average curvature. The usual weak field expansion on the other hand starts out in what has been described here as the 'rough' phase, and for which there appears to be no sensible definition for the path integral, in the sense that the curvatures are almost infinite and the volumes are almost zero, in units of the average edge length, and the lattice model therefore ceases to be a useful representation for the theory.

We have computed critical exponents associated with the non-trivial fixed point of four-dimensional euclidean gravity, and have shown that at the point where the average curvature vanishes, the curvature fluctuations diverge. Furthermore the values of the exponents computed show that the quantity $1/R_0M_0 \sim (4\lambda/k^2)_{eff}$ vanishes at the fixed point as well.

Many questions have remained open. It would be of interest to investigate how our results depend on the coupling a, and complete the picture for the phase diagram for pure gravity. For larger a one expects that the transition will move to larger values of k. The sensitivity of the results on the measure should be explored further. It would also be of interest to investigate these questions in the presence of matter fields, as well as for surfaces with boundaries.

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